MAT2705-01/02 13F Quiz 3 Print Name (Last, First) ____________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

typo! oops

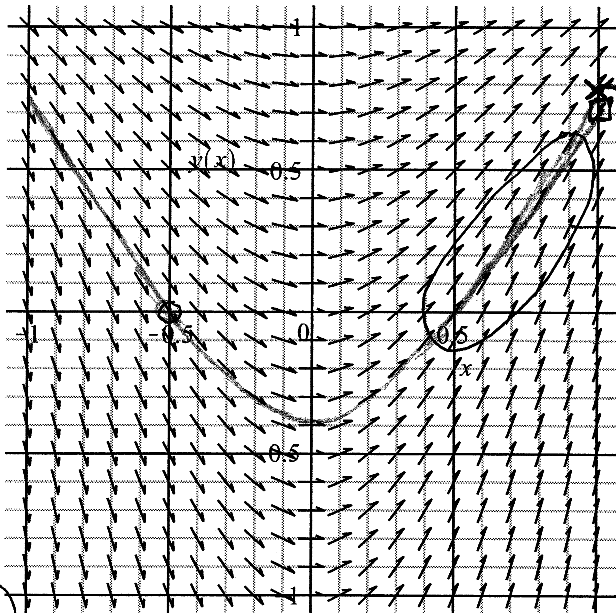
1. $\dfrac{dy}{dx} = \bar{}\,2xy + 3x\,,\quad y\left(-\dfrac{1}{2}\right) = 0\,,\ x \geq 0\,.$

a) Hand draw in the solution of this differential equation satisfying the initial condition on the associated direction field to the right. Put a circled dot at the point corresponding to the initial condition. Put a squared dot on the curve where $x = 1$. Estimate your approximate value of y ~~x=0~~ where this occurs. Don't change your estimated value after you later evaluate the solution exactly.

typo

b) Use the linear solution recipe to find the general solution of this differential equation. Simplify it and box it. [Do not use the separable technique.]

c) Find the solution of this differential equation which satisfies the given initial condition. Simplify it, combine exponentials, and box it.



calculated value 0.79

$y(1) \approx 0.7$

d) I drifted a bit to the right here bringing down my curve.

remark: $\dfrac{C}{e^{x^2}} = Ce^{-x^2}$ quotient function → composed exponential is simpler

$\int \ldots dx$ integral sign requires differential to indicate integration variable

d) Evaluate your solution at $x = 1$ numerically to 2 decimal places and mark the corresponding point on your graph with a visible **×** . Is this consistent with your part a) result? Explain.

e) Does your initial value problem solution agree with Maple ? If equivalent, show the equivalence. If not, can you find your mistake?

▶ solution

b) $e^{x^2}\left[\dfrac{dy}{dx} + 2x\,y = 3x\right] \longrightarrow \dfrac{d}{dx}\left(ye^{+x^2}\right) = 3xe^{+x^2}$

$e^{\int +2x\,dx = +x^2}$

$ye^{+x^2} = \int 3xe^{+x^2}dx = \int 3\,e^u\left(+\tfrac{1}{2}du\right) = +\tfrac{3}{2}\int e^u du$

$u = +x^2$

$\dfrac{du}{dx} = +2x \quad du = +2x\,dx$

$x\,dx = +\tfrac{1}{2}du$

$= +\tfrac{3}{2}e^u + C$

$= +\tfrac{3}{2}e^{+x^2} + C$

$\boxed{y = e^{-x^2}\left(+\tfrac{3}{2}e^{+x^2} + C\right) = +\tfrac{3}{2} + Ce^{-x^2}}$

c) $0 = y\left(-\tfrac{1}{2}\right) = +\tfrac{3}{2} + Ce^{-1/4}$

$C = -\tfrac{3}{2}e^{+1/4}$

$\boxed{y = +\tfrac{3}{2} - \tfrac{3}{2}e^{+1/4}e^{-x^2} = +\tfrac{3}{2} - \tfrac{3}{2}e^{-x^2+1/4}}$

$= \tfrac{3}{2}\left(-e^{-x^2+1/4} + 1\right)$

combine exponentials → not same, must combine exponentials

d) $y(1) = \tfrac{3}{2}\left(-e^{-1+1/4} + 1\right) = \tfrac{3}{2}\left(-e^{-3/4} + 1\right) \approx$

$\approx +0.79145$

$\approx \boxed{+0.79}$ ⟺ in graph: $\boxed{0.7}$

explanation: (see above)

e) Maple: $y(x) = \tfrac{3}{2} - \tfrac{3}{2}\dfrac{e^{-x^2}}{e^{-1/4}}$ combine!

> combine(%)

$y(x) = \tfrac{3}{2} - \tfrac{3}{2}e^{-x^2 + \frac{1}{4}}$