MAT2500-01/04 12S Quiz 8 Print Name (Last, First) ________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of each problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC).

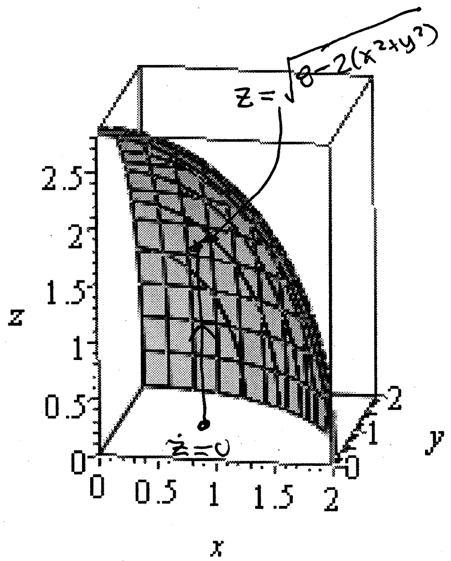
1. $M_{xy} = \iiint_R z \, dV$; $R$ is the region of the first octant bounded by the coordinate planes $x=0, y=0, z=0$ and the ellipsoid $2x^2 + 2y^2 + z^2 = 8$

a) Iterate this triple integral in Cartesian coordinates in the order ∭ ... *dz dy dx*, then use Maple to evaluate the result exactly (don't use decimal numbers anywhere). Support your result with a diagram of the outer double integral region of integration in the *xy*-plane showing one typical line segment cross-section with its endpoints labeled, and "shade in" the region with equally spaced such cross-section line segments, while stating the floor and ceiling function graphs which bound the region in the *z* direction.

b) Re-express the outer double integral (*dy dx*) in polar coordinates, and evaluate by hand easily this triple integral. In expressing this polar coordinate double integral, make a diagram of the region of integration in the *xy*-plane showing one typical radial cross-section with its endpoints labeled, and "shade in" the region with equally spaced radial cross-section line segments. (the triple integral)

c) Now use Maple to evaluate this last integral exactly (~~be sure to show the step-by-step evaluation process~~ for ~~triple integral~~) and compare to part a). They must agree of course.

d) **Optional.** Ignore this please: use polar coordinates to evaluate the volume $V$ of $R$, and then evaluate the ratio: $\bar{z} = M_{xy}/V$ numerically to get the height of the centroid of this solid region.

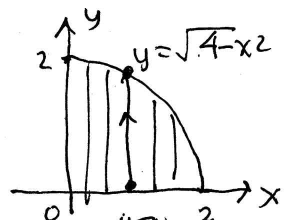



a) $2x^2+2y^2+z^2=8 \rightarrow z=0 \quad 2x^2+2y^2=8$

$z=\pm\sqrt{8-2x^2-2y^2}$ $\qquad x^2+y^2=4$

$r=2.$

$y=\sqrt{4-x^2} \geq 0$



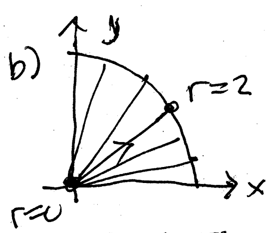

Floor: $z=0$

Ceiling: $z=\sqrt{8-2(x^2+y^2)}$

$$\boxed{M_{xy} = \int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^{\sqrt{8-2(x^2+y^2)}} z \, dz \, dy \, dx \overset{\text{Maple}}{=} 2\pi}$$

## ▶ solution

b)




$\theta = 0..\pi/2$

$dA = r\,dr\,d\theta$

$$M_{xy} = \int_0^{\frac{\pi}{2}} \int_0^2 \int_0^{\sqrt{8-2r^2}} z(r)\,dz\,dr\,d\theta \overset{\text{maple}}{=} 2\pi$$

$$r\frac{z^2}{2}\Big|_{z=0}^{z=8-2r^2} = \frac{r}{2}(8-2r^2) = 4r-r^3$$

$$= \int_0^{\frac{\pi}{2}} d\theta \int_0^2 4r-r^3\,dr = \frac{\pi}{2}\left(2r^2-\frac{r^4}{4}\right)\Big|_0^2 = \frac{\pi}{2}(8-4) = \boxed{2\pi} \checkmark$$

Maple agrees also with this triple integral value.

d) $$V = \int_0^{\frac{\pi}{2}} \int_0^2 \int_0^{\sqrt{8-2r^2}} 1\, r\,dz\,dr\,d\theta = \int_0^{\frac{\pi}{2}} d\theta \int_0^2 r(8-2r^2)^{1/2}\,dr \quad \int u^{\frac{1}{2}}\left(\frac{du}{-4}\right) = -\frac{1}{4}\frac{u^{3/2}}{3/2}$$

$zr\Big|_{z=0}^{z=\sqrt{8-2r^2}}$ $\qquad u \quad du=-4r\,dr$

$$= \frac{\pi}{2}\left(-\frac{1}{4}\frac{2}{3}\right)(8-2r^2)^{3/2}\Big|_{r=0}^{r=2} = \frac{\pi}{12}8^{3/2} = \frac{\pi}{3}2^{\frac{1}{2}} = \frac{\pi 4\sqrt{2}}{3}$$

$$\bar{z} = \frac{M_{xy}}{V} = \frac{2\pi}{\frac{4\pi}{3}\sqrt{2}} = \boxed{\frac{3}{2\sqrt{2}} = \frac{3}{4}\sqrt{2}} \approx 1.06$$

reasonable