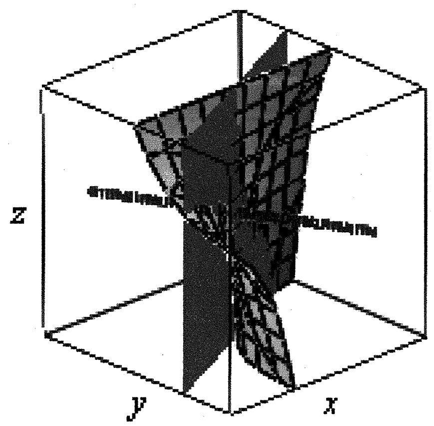
MAT2500-01/04 12S Quiz 6 Print Name (Last, First) ____________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of each problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC).



1. $f(x, y, z) = y^2 e^{xyz}$

a) Write an equation for the level surface of this function passing through (0,1,-1).

b) Evaluate the gradient vector field and its value at (0,1,-1), identifying it with its proper symbol.

c) In what direction (unit vector) at (0,1,-1) does f increase the most rapidly and what is its rate of change in that direction?

d) Write an equation for the normal line to the level surface through (0,1,-1), namely the line through this point which is aligned with the gradient vector there. At what point $\langle x_0, 0, z_0\rangle$ does the normal line intersect the *x-z* plane?

e) What is the directional derivative of f at (0,1,-1) in the same direction as the vector $\langle 3, 12, 4\rangle$? Identify it by its proper symbol. Is the function increasing or decreasing in this direction?

f) **Optional. Only for students who finish ahead of time.**
Are there other points where the gradient vector is parallel to this one at (0,1,-1)?

▶ solution

a) $f(0,1,-1) = 1^2 e^{0(1)(-1)} = 1$

level surface: $\boxed{y^2 e^{xyz} = 1}$

b) $f_x = \frac{\partial}{\partial x}(y^2 e^{xyz}) = y^2 e^{xyz} \frac{\partial}{\partial x}(xyz) = y^3 z e^{xyz}$

$f_y = \frac{\partial}{\partial y}(y^2 e^{xyz}) = 2y e^{xyz} + y^2 e^{xyz} \underbrace{\frac{\partial}{\partial y}(xyz)}_{xz}$

$= (2y + xy^2 z) e^{xyz}$

$f_z = \frac{\partial}{\partial z}(y^2 e^{xyz}) = y^2 e^{xyz} \frac{\partial}{\partial z}(xyz) = xy^3 e^{xyz}$

$\vec\nabla f = \langle f_x, f_y, f_z\rangle = \boxed{\langle y^3 z,\ 2y + xy^2 z,\ xy^3\rangle e^{xyz}}$

$= y e^{xyz} \langle y^2 z,\ 2 + xyz,\ xy^2\rangle$

$\vec\nabla f(0,1,-1) = \langle 1^3(-1),\ 2(1) + 0,\ 0\rangle e^0 = \boxed{\langle -1, 2, 0\rangle}$

c) $\widehat{\nabla f}(0,1,-1) = \boxed{\frac{\langle -1,2,0\rangle}{\sqrt5}}$

$\max(D_{\hat u} f(0,1,-1)) = |\vec\nabla f(0,1,-1)| = \boxed{\sqrt5}$

d) $\langle y^2 z, 2 + xyz, xy^2\rangle\Big|_{x=0,\ y=1,\ z=-1} = \langle -1, 2, 0\rangle = \vec n$

$\vec r_0 = \langle 0, 1, -1\rangle$

$\vec r = \vec r_0 + t\vec n = \langle 0,1,-1\rangle + t\langle -1,2,0\rangle = \boxed{\langle -t, 1+2t, -1\rangle = \langle x, y, z\rangle}$

$0 = y = 1 + 2t \to t = -\frac12 \to x = -(-\frac12) = \frac12,\ z = -1$ $\boxed{\text{point } (\tfrac12, 0, -1)}$

e) $\vec u = \langle 3, 12, 4\rangle$

$|\vec u| = \sqrt{3^2 + 4^2 + 12^2} = \sqrt{5^2 + 12^2} = \sqrt{13^2} = 13$

$\hat u = \frac{1}{13}\langle 3, 12, 4\rangle$

$D_{\hat u} f(0,1,-1) = \hat u \cdot \vec\nabla f(0,1,-1)$

$= \frac{1}{13}\langle 3,12,4\rangle \cdot \langle -1,2,0\rangle = \frac{-3+24}{13}$

$= \boxed{21/13} > 0$ so $\boxed{\text{increasing}}$

f) $\vec\nabla f(xyz) \propto \langle -1, 2, 0\rangle$

$\langle y^2 z, 2 + xyz, xy^2\rangle = k\langle -1, 2, 0\rangle$

$y^2 z = -k,$

$2 + xyz = 2k$

$xy^2 = 0 \to \boxed{x=0}$ or $y = 0$

$y = 0 \to k = 0 \to 2 + 0 = 0$

$x = 0 \to 2 + 0 = 2k \to k = 1 \to y^2 z = -1$

$\boxed{z = -1/y^2}$

$\boxed{\text{all points on the curve } x=0,\ z=-1/y^2 \text{ have a gradient parallel to } \vec\nabla f(0,1,-1)}$