MAT2500-01/04 12s Quiz 4 Take Home Print Name (Last, First) ________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC).

1. $f(x, y) = e^{xy}\sin(x+y)$

a) Evaluate f and the first partial derivatives of f at $(x, y) = (1, \pi/2)$ using proper notation for all derivatives evaluated in the process.

b) Evaluate $f_{xy}(x, y)$ and its value at $(1, \pi/2)$.

2. a) If $w = \sqrt{u^2 - v^2}$, what is the domain of this function? [Make a crude sketch shading in the domain.]

b) Evaluate $\dfrac{\partial^2 w}{\partial u\, \partial v}$.

3. The heat index $I = f(T, H)$ is the perceived Farenheit temperature in degrees when the actual temperature is T degrees Farenheit but the relative humidity in percent is H. Translate the mathematical condition $105 = f(92, 60)$ into a complete English sentence.

▶ solution

$$f_{xy}(1,\tfrac{\pi}{2}) = e^{\frac{\pi}{2}}\left(\tfrac{\pi}{2}\sin(1+\tfrac{\pi}{2}) + (1+\tfrac{\pi}{2})\cos(1+\tfrac{\pi}{2})\right)$$

$$f_{xy}(x,y) = e^{xy}[xy\sin(x+y) + (x+y)\cos(x+y)]$$ if wish to simplify further (optional)

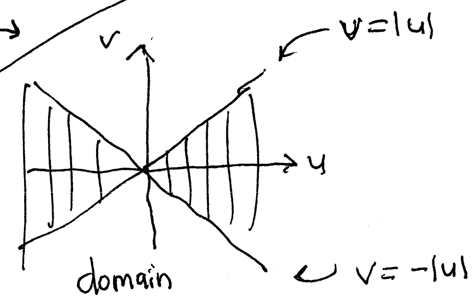
① a) $f(x,y) = e^{xy}\sin(x+y)$.

$$f_x(x,y) = \frac{\partial}{\partial x}(e^{xy}\sin(x+y)) = (e^{xy}y)\sin(x+y) + e^{xy}(\cos(x+y)(1+0))$$
$$= \boxed{e^{xy}(y\sin(x+y) + \cos(x+y))}$$

$$f_y(x,y) = \frac{\partial}{\partial y}(e^{xy}\sin(x+y)) = (e^{xy}x)\sin(x+y) + e^{xy}(\cos(x+y)(0+1))$$
$$= \boxed{e^{xy}(x\sin(x+y) + \cos(x+y))}$$

$$f_{xy}(x,y) = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial y}\left(e^{xy}(y\sin(x+y) + \cos(x+y))\right)$$
$$= (e^{xy}x)(y\sin(x+y) + \cos(x+y))$$
$$+ e^{xy}[1\sin(x+y) + y\cos(x+y)(0+1) - \underbrace{\sin(x+y)(0+1)}_{0}]$$
$$= \boxed{e^{xy}(xy\sin(x+y) + y\cos(x+y) + x\cos(x+y))}$$

$$f(1,\tfrac{\pi}{2}) = e^{1(\pi/2)}\sin(1+\tfrac{\pi}{2}) = e^{\frac{\pi}{2}}\sin(1+\tfrac{\pi}{2})$$
$$f_x(1,\tfrac{\pi}{2}) = e^{\pi/2}(\tfrac{\pi}{2}\sin(1+\tfrac{\pi}{2}) + \cos(1+\tfrac{\pi}{2}))$$
$$f_y(1,\tfrac{\pi}{2}) = e^{\pi/2}(\sin(1+\tfrac{\pi}{2}) + \cos(1+\tfrac{\pi}{2}))$$
$$f_{xy}(1,\tfrac{\pi}{2}) = e^{\pi/2}(\tfrac{\pi}{2}\sin(1+\pi/2) + \pi/2\cos(1+\pi/2) + \cos(1+\pi/2))$$



② a) $u^2 - v^2 \geq 0$, $u^2 \geq v^2$, $|u| \geq |v|$

b) $$\frac{\partial w}{\partial v} = \frac{\partial}{\partial v}(u^2-v^2)^{1/2} = \frac{1}{2}(u^2-v^2)^{-1/2}(0-2v) = \frac{-v}{(u^2-v^2)^{1/2}}$$

$$\frac{\partial^2 w}{\partial u\,\partial v} = \frac{\partial}{\partial u}\left(\frac{-v}{(u^2-v^2)^{1/2}}\right) = \cancel{(u^2-v^2)^{1/2}(-1}\ -v\frac{\partial}{\partial u}(u^2-v^2)^{-1/2} = -v\left(-\frac{1}{2}\right)(u^2-v^2)^{-3/2}(2u)$$
$$= \boxed{\frac{uv}{(u^2-v^2)^{3/2}}}$$

③ If the actual temperature is 92° but the humidity is 60%, then it feels like 105°