

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

Given the vector-valued function $\vec{r}(t) = \left\langle \frac{t^2}{2}, \frac{t^4}{4}, \frac{t^6}{6} \right\rangle$, (no credit for unidentified expressions in your responses):

- a) Evaluate $\vec{r}'(t)$, $\vec{r}''(t)$, $|\vec{r}'(t)|$, $\hat{T}(t)$ and remember to simplify your results.
- b) Evaluate $\vec{r}(1)$, $\vec{r}'(1)$, $\vec{r}''(1)$, $\hat{T}(1)$ and remember to simplify your results.
- c) Evaluate the exact angle θ in radians between $\vec{r}'(1)$ and $\vec{r}''(1)$ and a single decimal place approximation in degrees.
- d) Evaluate the vector \vec{w} which is the vector projection of $\vec{r}''(1)$ orthogonal (perpendicular!) to $\vec{r}'(1)$.

► solution

a) $\vec{r} = \left\langle \frac{1}{2}t^2, \frac{1}{4}t^4, \frac{1}{6}t^6 \right\rangle$
 $\vec{r}' = \langle t, t^3, t^5 \rangle$
 $\vec{r}'' = \langle 1, 3t^2, 5t^4 \rangle$

$|\vec{r}'| = \sqrt{t^2 + t^6 + t^{10}} = |t| \sqrt{1 + t^4 + t^8}$

$\hat{T} = \frac{\vec{r}'}{|\vec{r}'|} = \frac{\langle t, t^3, t^5 \rangle}{\sqrt{t^2 + t^6 + t^{10}}} = \frac{t \langle 1, t^2, t^4 \rangle}{|t| \sqrt{1 + t^4 + t^8}} = \frac{\text{sgn}(t) \langle 1, t^2, t^4 \rangle}{\sqrt{1 + t^4 + t^8}}$
 but not in simplest form!
 common factors.

b) $\vec{r}(1) = \left\langle \frac{1}{2}, \frac{1}{4}, \frac{1}{6} \right\rangle$
 $\vec{r}'(1) = \langle 1, 1, 1 \rangle$
 $\vec{r}''(1) = \langle 1, 3, 5 \rangle$
 $\hat{T}(1) = \frac{\langle 1, 1, 1 \rangle}{\sqrt{3}}$

$\vec{r}''(1) = \frac{\langle 1, 3, 5 \rangle}{\sqrt{1+9+25}} = \frac{\langle 1, 3, 5 \rangle}{\sqrt{35}}$

c) $\cos \theta = \hat{r}'(1) \cdot \hat{r}''(1) = \hat{T}(1) \cdot \vec{r}''(1) = \frac{\langle 1, 1, 1 \rangle}{\sqrt{3}} \cdot \frac{\langle 1, 3, 5 \rangle}{\sqrt{35}} = \frac{1+3+5}{\sqrt{3}\sqrt{35}} = \frac{9}{\sqrt{3}\sqrt{35}}$

$\theta = \arccos\left(\frac{9}{\sqrt{3}\sqrt{35}}\right) = \arccos\left(\frac{3\sqrt{3}}{\sqrt{35}}\right) \approx 28.6^\circ$

d) $\vec{r}''(1) \cdot \hat{T}(1) = \langle 1, 3, 5 \rangle \cdot \frac{\langle 1, 1, 1 \rangle}{\sqrt{3}} = \frac{9}{\sqrt{3}} = 3\sqrt{3}$

$[\vec{r}''(1)]_{\parallel} = 3\sqrt{3} \hat{T}(1) = 3\sqrt{3} \frac{\langle 1, 1, 1 \rangle}{\sqrt{3}} = 3 \langle 1, 1, 1 \rangle$

$\vec{w} = [\vec{r}''(1)]_{\perp} = \vec{r}''(1) - [\vec{r}''(1)]_{\parallel} = \langle 1, 3, 5 \rangle - \langle 3, 3, 3 \rangle = \langle -2, 0, 2 \rangle = \vec{w}$

bonus remark:

sizes of par/perp vector components give geometry of projection rectangle

$\vec{a} = \vec{r}''(1)$

perp: $|\vec{w}| = 2\sqrt{2} \approx 2.828$

par: $3\sqrt{3} \approx 5.196$

$|\vec{r}''(1)| = \sqrt{35} \approx 5.916$

