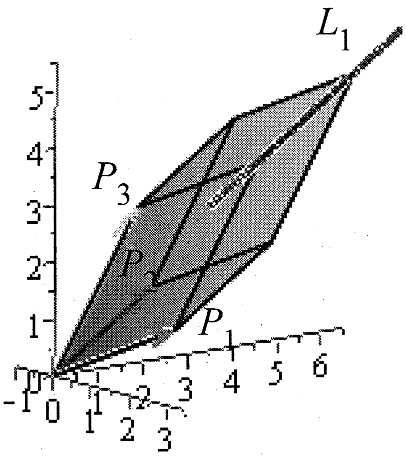
MAT2500-01/04 12S Quiz 2 Print Name (Last, First) ______________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

Given three points $P_1(2,1,1)$, $P_2(-1,3,1)$, $P_3(1,1,3)$ and the parallelopiped formed from their three position vectors $\vec{r}_1, \vec{r}_2, \vec{r}_3$. [Note $\vec{r}_1, \vec{r}_3$ form the front face of this object, with $\vec{r}_2$ extending backwards from this face.]



a) Write the parametrized equations of the line L_1 through the top right edge of the parallelopiped as shown. [What is the simplest position vector of a point on the line? What is the orientation of the line?]

b) Find a normal vector $\vec{n}$ for the for the plane $\mathbb{P}_{top}$ which contains the top face of the parallelepiped shown in the figure.

c) Write the simplified equation for this plane.

d) Find the scalar projection h of $\vec{r}_3$ along $\vec{n}$.

e) Evaluate the area A of the bottom face of the parallelogram, a parallelogram formed by the edges $\vec{r}_1, \vec{r}_2$

f) Does the volume $V=|h|\,A$ of the parallelopiped equal the triple scalar product $|\vec{r}_3 \cdot (\vec{r}_1 \times \vec{r}_2)|$ as it should?

▶ solution

a) front top corner: $\vec{r}_1+\vec{r}_3 = \langle 2,1,1\rangle + \langle 1,1,3\rangle = \langle 2+1, 1+1, 1+3\rangle = \langle 3,2,4\rangle$

orientation of L_1: $\vec{r}_2$

L_1: $\underset{\langle x,y,z\rangle}{\vec{r}} = (\vec{r}_1+\vec{r}_3) + t\vec{r}_2 = \langle 3,2,4\rangle + t\langle -1,3,1\rangle = \langle 3-t, 2+3t, 4+t\rangle$

$\boxed{\langle x,y,z\rangle = \langle 3-t, 2+3t, 4+t\rangle}$ vector form

$\boxed{x=3-t,\ y=2+3t,\ z=4+t}$ scalar form

b) $\mathbb{P}_{top}$ contains directions of $\vec{r}_1$ and $\vec{r}_2$ so a normal is:

$\vec{n} = \vec{r}_1\times\vec{r}_2 = \langle 2,1,1\rangle \times \langle -1,3,1\rangle = \begin{vmatrix} i & j & k \\ 2 & 1 & 1 \\ -1 & 3 & 1 \end{vmatrix} = \langle 1(1)-1(3),\ 1(-1)-2(1),\ 2(3)-1(-1)\rangle = \boxed{\langle -2,-3,7\rangle}$

note $|\vec{n}| = \sqrt{4+9+49} = \sqrt{62}$

c) We can use either $\vec{r}_3$ or $\vec{r}_1+\vec{r}_3$ as the position vector of a point on this plane — it won't matter to the final result:

$0 = \vec{n}\cdot(\vec{r}-\vec{r}_0) = \langle -2,-3,7\rangle \cdot \langle x-1, y-1, z-3\rangle = -2(x-1)-3(y-1)+7(z-3)$

$= -2x-3y+7z+\underbrace{2+3-21}_{-16}$

$\boxed{2x+3y-7z=-16 \quad \text{or} \quad -2x-3y+7z=16}$

d) $h = \hat{n}\cdot\vec{r}_3 = \frac{1}{\sqrt{62}}\langle -2,-3,7\rangle\cdot\langle 1,1,3\rangle = \frac{-2-3+21}{\sqrt{62}} = \boxed{\frac{16}{\sqrt{62}}}$ (or $-\frac{16}{\sqrt{62}}$ if we had used $\vec{n} = \vec{r}_2\times\vec{r}_1$)

e) $A = |\vec{r}_1\times\vec{r}_2| = |\vec{n}| = \boxed{\sqrt{62}}$

f) $V = \left|\frac{16}{\sqrt{62}}\right|\sqrt{62} = 16$ $\quad \vec{r}_3\cdot(\vec{r}_1\times\vec{r}_2) = \langle 1,1,3\rangle\cdot\langle -2,-3,7\rangle = -2-3+21 = 16$ ✓ they agree