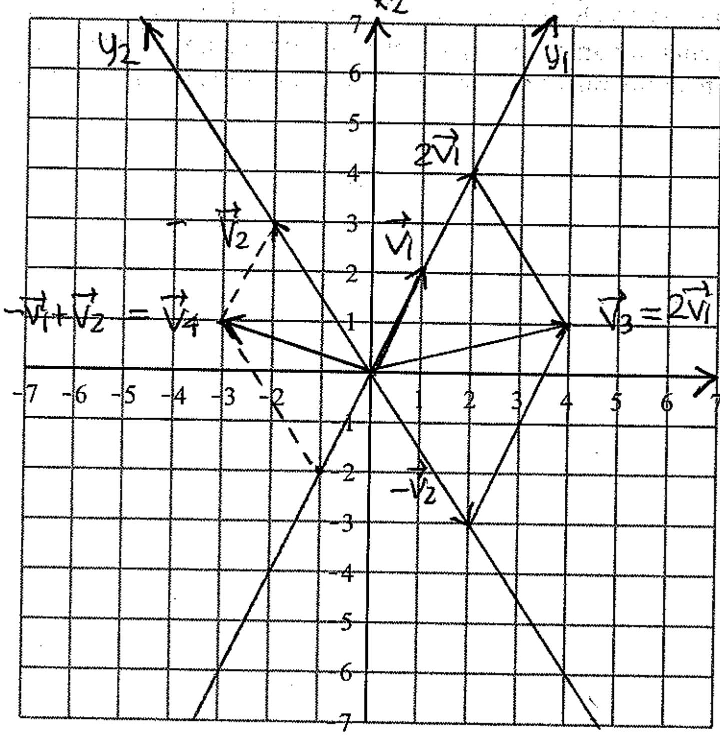
MAT 2705-01/02 12F Test 2 Answers

a)



e) $\langle -3,1\rangle = -\vec{v}_1+\vec{v}_2 = \vec{v}_4$ $\qquad \vec{v}_3 = 2\vec{v}_1 - \vec{v}_2 = \langle 4,1\rangle$

b) $y_1\begin{bmatrix}1\\2\end{bmatrix} + y_2\begin{bmatrix}-2\\3\end{bmatrix} = \begin{bmatrix}4\\1\end{bmatrix}$

$\underbrace{\begin{bmatrix}1&-2\\2&3\end{bmatrix}}_{B}\underbrace{\begin{bmatrix}y_1\\y_2\end{bmatrix}}_{\vec{y}} = \begin{bmatrix}4\\1\end{bmatrix}$

$\vec{y} = B^{-1}\begin{bmatrix}4\\1\end{bmatrix} = \frac{1}{3+4}\begin{bmatrix}3&2\\-2&1\end{bmatrix}\begin{bmatrix}4\\1\end{bmatrix}$

$= \frac{1}{7}\begin{bmatrix}12+2\\-8+1\end{bmatrix} = \frac{1}{7}\begin{bmatrix}14\\-7\end{bmatrix} = \begin{bmatrix}2\\-1\end{bmatrix} = \begin{bmatrix}y_1\\y_2\end{bmatrix}$

so $\boxed{\begin{bmatrix}4\\1\end{bmatrix} = 2\begin{bmatrix}1\\2\end{bmatrix} - \begin{bmatrix}-2\\3\end{bmatrix}}$

c) $= \begin{bmatrix}2&2\\4&-3\end{bmatrix} = \begin{bmatrix}4\\1\end{bmatrix}$ ✓

d) $(y_1,y_2) = (2,-1)$ checks!

e) $-\begin{bmatrix}1\\2\end{bmatrix} + \begin{bmatrix}-2\\3\end{bmatrix} = \begin{bmatrix}-1-2\\-2+3\end{bmatrix} = \begin{bmatrix}-3\\1\end{bmatrix}$

checks with graphical readout ✓

② a) $A = \begin{bmatrix}2&-2&6&2\\1&3&-1&9\\-3&1&-7&-7\\1&2&0&7\end{bmatrix}$ $\quad \vec{b} = \begin{bmatrix}-4\\2\\4\\0\end{bmatrix}$

$\langle A|\vec{b}\rangle \xrightarrow{\text{rref}} \begin{bmatrix}1&0&2&3&0\\0&1&-1&2&0\\0&0&0&0&1\\0&0&0&0&0\end{bmatrix}$

implies $0 = 1$ so inconsistent system, no soln

② a) no linear combination of the columns equals this $\vec{b}$

b) $\langle A|\vec{b}\rangle \xrightarrow{\text{rref}}$ $\begin{array}{ccccc} L & L & F & F & \\ x_1 & x_2 & x_3 & x_4 & \end{array}$ $\begin{bmatrix}1&0&2&3&-1\\0&1&-1&2&1\\0&0&0&0&0\\0&0&0&0&0\end{bmatrix}$

$\vec{b} = \begin{bmatrix}-4\\2\\4\\0\end{bmatrix}$

$x_3 = t_1 \qquad x_1 + 2t_1 + 3t_2 = -1 \qquad x_1 = -1-2t_1-3t_2$

$x_4 = t_2 \qquad x_2 - t_1 + 2t_2 = 1 \qquad x_2 = 1+t_1-2t_2$

$\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} = \begin{bmatrix}-1-2t_1-3t_2\\1+t_1-2t_2\\t_1\\t_2\end{bmatrix}$

yes $\vec{v}_5$ does lie in the span of $\{\vec{v}_1,\vec{v}_2,\vec{v}_3,\vec{v}_4\}$.

so $\boxed{\begin{bmatrix}-4\\2\\4\\1\end{bmatrix} = (-1-2t_1-3t_2)\begin{bmatrix}2\\1\\-3\\1\end{bmatrix} + (1+t_1-2t_2)\begin{bmatrix}-2\\3\\1\\2\end{bmatrix} + t_1\begin{bmatrix}6\\-1\\-7\\0\end{bmatrix} + t_2\begin{bmatrix}2\\9\\-7\\7\end{bmatrix}}$

c) $\langle A|\vec{0}\rangle \xrightarrow{\text{rref}} \begin{bmatrix}1&0&2&3&0\\0&1&-1&2&0\\0&0&0&0&0\\0&0&0&0&0\end{bmatrix}$ $\begin{array}{cccc} L & L & F & F\end{array}$ $\rightarrow$ soln = hom part of previous soln

$\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} = \begin{bmatrix}-2t_1-3t_2\\t_1-2t_2\\t_1\\t_2\end{bmatrix} = t_1\underbrace{\begin{bmatrix}-2\\1\\1\\0\end{bmatrix}}_{\text{“}\vec{u}_1\text{”}} + t_2\underbrace{\begin{bmatrix}-3\\-2\\0\\1\end{bmatrix}}_{\vec{u}_2\text{”}}$ $\quad$ d) $\{\vec{u}_1,\vec{u}_2\}$ is a basis

②d) $\{\vec{u}_1,\vec{u}_2\} = \{\langle -2,1,1,0\rangle, \langle -3,-2,0,1\rangle\}$

e) $\vec{u}_1$: $-2\vec{v}_1 + \vec{v}_2 + \vec{v}_3 = \vec{0}$

$\vec{u}_2$: $-3\vec{v}_1 - 2\vec{v}_2 + \vec{v}_4 = \vec{0}$

f) span$\{\vec{v}_1,\vec{v}_2,\vec{v}_3,\vec{v}_4\}$ = 2-dimensional $\boxed{\text{plane}}$ in $\mathbb{R}^4$.

basis $\{\vec{v}_1,\vec{v}_2\}$ or any 2 of the 4 vectors

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① a) $\boxed{\vec v_3 = 3\vec v_1 - 2\vec v_2}$ has coordinates $\boxed{\langle y_1, y_2\rangle = \langle 3,-2\rangle}$ wrt the new basis $\{\vec v_1, \vec v_2\}$.

Tip to tail vector addition with the key parallelogram makes this obvious.

b) $y_1\begin{bmatrix}1\\1\end{bmatrix} + y_2\begin{bmatrix}3\\-1\end{bmatrix} = \boxed{\begin{bmatrix}1&3\\1&-1\end{bmatrix}\begin{bmatrix}y_1\\y_2\end{bmatrix} = \begin{bmatrix}-3\\5\end{bmatrix}}$

$$\begin{bmatrix}y_1\\y_2\end{bmatrix} = \underbrace{\frac{1}{-1-3}\begin{bmatrix}-1&-3\\-1&1\end{bmatrix}}_{\text{inverse coeff matrix}}\begin{bmatrix}-3\\5\end{bmatrix} = \frac14\begin{bmatrix}1&3\\1&-1\end{bmatrix}\begin{bmatrix}-3\\5\end{bmatrix}$$

$$= \frac14\begin{bmatrix}-3+3(5)\\-3-5\end{bmatrix} = \frac14\begin{bmatrix}12\\-8\end{bmatrix} = \begin{bmatrix}3\\-2\end{bmatrix}$$

so $\boxed{\langle -3,5\rangle = 3\langle 1,1\rangle - 2\langle 3,-1\rangle}$

c) $= \langle 3,3\rangle - \langle 6,-2\rangle$

$= \langle -3,5\rangle$ ✓

d) They agree! Yay!

e) $\vec v_4 = -4\vec v_1 + \vec v_2 = -4\langle 1,1\rangle + \langle 3,-1\rangle$

$= \langle -4,-4\rangle + \langle 3,-1\rangle = \underbrace{\boxed{\langle -1,-5\rangle} = \boxed{\langle x_1,x_2\rangle}}_{\text{agrees with grid readout}}$

② $A = \begin{bmatrix}3&-1&1&0\\2&3&8&1\\-1&4&7&-2\\4&2&8&3\end{bmatrix} \xrightarrow[\text{maple}]{\text{rref}} \begin{bmatrix}1&0&1&0\\0&1&2&0\\0&0&0&1\\0&0&0&0\end{bmatrix}$ so $\vec v_3 = \vec v_1 + 2\vec v_2$ but $\{\vec v_1, \vec v_2, \vec v_3\}$ are lin ind.

a) $\langle A \mid \langle 1,7,9,5\rangle\rangle \xrightarrow[\text{maple}]{\text{rref}} \begin{array}{c}\begin{bmatrix}1&0&1&0&1\\0&1&2&0&2\\0&0&0&1&-1\\0&0&0&0&0\end{bmatrix}\\ \begin{array}{cccc}x_1&x_2&x_3&x_4\\ L&L&F&L\end{array}\end{array}$

so $\langle 1,7,9,5\rangle = \vec v_1 + 2\vec v_2 - \vec v_3$ ← particular soln

gen soln: $x_3 = t$

$x_1 + x_3 = 1$ $\quad x_1 = 1-t$

$x_2 + 2x_3 = 2$ $\quad x_2 = 2-2t$

$x_4 = -1$ $\quad x_3 = t$

$\quad x_4 = -1$

$$\vec x = \begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} = \begin{bmatrix}1-t\\2-2t\\t\\1\end{bmatrix} = \begin{bmatrix}1\\2\\0\\1\end{bmatrix} + t\begin{bmatrix}-1\\-2\\1\\0\end{bmatrix}$$

So $\boxed{\langle 1,7,9,5\rangle = (1-t)\vec v_1 + 2(1-t)\vec v_2 + t\vec v_3 + \vec v_4}$

②b) $\langle A \mid \langle 2,-3,1,4\rangle\rangle \xrightarrow[\text{maple}]{\text{rref}} \begin{bmatrix}1&0&1&0&0\\0&1&2&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix} \leftarrow 0=1$ inconsistent system

so this vector does not lie in span$(\{\vec v_1, \vec v_2, \vec v_3, \vec v_4\})$

c) homogeneous soln: $\boxed{\vec x = \langle -t, -2t, t, 0\rangle}$

okay, following instructions

$\begin{bmatrix}3&-1&1&0&0\\2&3&8&1&0\\-1&4&7&-2&0\\4&2&8&3&0\end{bmatrix} \xrightarrow[\text{maple}]{\text{rref}} \begin{bmatrix}1&0&1&0&0\\0&1&2&0&0\\0&0&0&1&0\\0&0&0&0&0\end{bmatrix}$

$\begin{array}{cccc}x_1&x_2&x_3&x_4\\ L&L&F&L\end{array}$

$\begin{bmatrix}3&-1&1&0\\2&3&8&1\\-1&4&7&-2\\4&2&8&3\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} = \begin{bmatrix}0\\0\\0\\0\end{bmatrix}$

d) $\vec u = \langle -1,-2,1,0\rangle$, $\{\vec u\}$ is a basis of the hom. soln. space.

e) $\boxed{-\vec v_1 - 2\vec v_2 + \vec v_3 = \vec 0}$

f) only 3 ind vectors so it is a $\boxed{\text{hyperplane}}$

$\boxed{\text{basis: } \{\vec v_1, \vec v_2, \vec v_4\}}$

(leading columns of A)