

MAT 2500-01/02 12F Test 1 Answers

① a)  $\frac{dx}{dt} + \frac{5}{20+5t} x = 10$

b)  $x(t) = \frac{40t + 5t^2 + 4}{4+t}$

d)  $V = 20 + 5t = 100$   
 $5t = 80, t = \frac{80}{5} = 16 = T$

$x(16) \approx 95$

e)  $\left[ \frac{dx}{dt} + \frac{5}{20+5t} x = 10 \right]$   
 $\int \frac{5}{20+5t} dt = 5 \ln|20+5t|$   
 $e^{\frac{5}{20+5t}} = (20+5t)$

$\frac{d}{dt}(x(20+5t)) = 10(20+5t)$

$x(20+5t) = \int 10(20+5t) dt = 2 \int u du$   
 $= 2 \frac{u^2}{2} + C = (20+5t)^2 + C$

$x = \frac{(20+5t)^2 + C}{20+5t} = \frac{20+5t + \frac{C}{20+5t}}{20+5t}$

or:  $x(20+5t) = \int 10(20+5t) dt = 10(20t + 5t^2/2) + C_2$   
 $= 200t + 25t^2 + C_2$

$x = \frac{200t + 25t^2 + C_2}{20+5t}$   $\leftrightarrow$   $C_2 = C + 400$

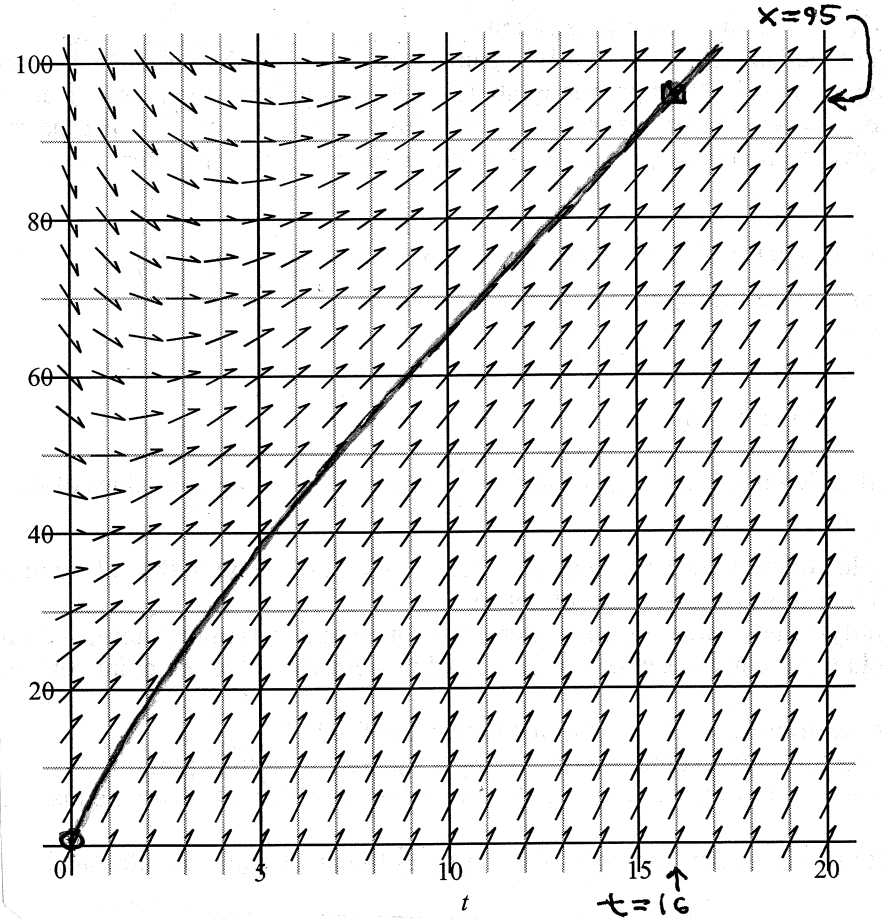
f)  $1 = x(0) = \frac{C_2}{20} = 20 + \frac{C}{20} \rightarrow C_2 = 20$   
 $C = 400 + 20 = 380$

$x = 20 + 5t - \frac{380}{20+5t} = \frac{200t + 25t^2 + 200}{20+5t}$   
 $= 20 + 5t - \frac{76}{4+t} = \frac{40t + 5t^2 + 4}{4+t}$

g)  $x(16) = 20 + 80 - \frac{380}{20+80} = 100 - 3.9 = 96.2$

very close to estimate of 95 which was already a hair low

h) Maple combines fractions here - easy to do to verify same



② a)  $\frac{dx}{dt} + \frac{5}{20+5t} x = 0$

$\frac{dx}{dt} = -\frac{5}{20+5t} x$

$\int \frac{dx}{x} = \int \frac{5}{20+5t} dt$

$\ln|x| = -5 \frac{1}{5} \ln|20+5t| + C_1$

$|x| = (20+5t)^{-1} e^{C_1}$

$x = \pm \frac{e^a}{20+5t} = \frac{C}{20+5t}$

b)  $1 = x(0) = \frac{C}{20} \rightarrow C = 20$

$x = \frac{20}{20+5t} = \frac{4}{4+t}$

c)  $x(16) = \frac{4}{4+16} = \frac{4}{20} = \frac{1}{5} = 0.2 \text{ lbs}$

d) Maple gives second simplified form.