Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

1. $\dfrac{dy}{dx} = -2y + 3e^{-3x}$, $y(0) = 1$, $x \geq 0$.

a) Hand draw in the solution of this differential equation satisfying the initial condition on the associated direction field to the right. Put a circled dot at the point corresponding to the initial condition. Put a squared dot on the curve at $x = 1.5$. Estimate your approximate value of $y(1.5)$.

b) Use the linear solution recipe to find the general solution of this differential equation. Simplify it and box it.

c) Find the solution of this differential equation which satisfies the given initial condition. Simplify it and box it.

d) Evaluate y(1.5) numerically to 2 decimal places and mark the corresponding point on your graph with a triangled dot. Is this consistent with your part a) result? Explain.

e) Does your initial value problem solution agree with Maple (which requires Expand to multiply out, and Simplify, Simplify to simplify)? If not, can you find your mistake? If so, show the equivalence of your solution with Maple's. Did you simplify your solution as requested before comparing it to Maple's?

a)

y(x)
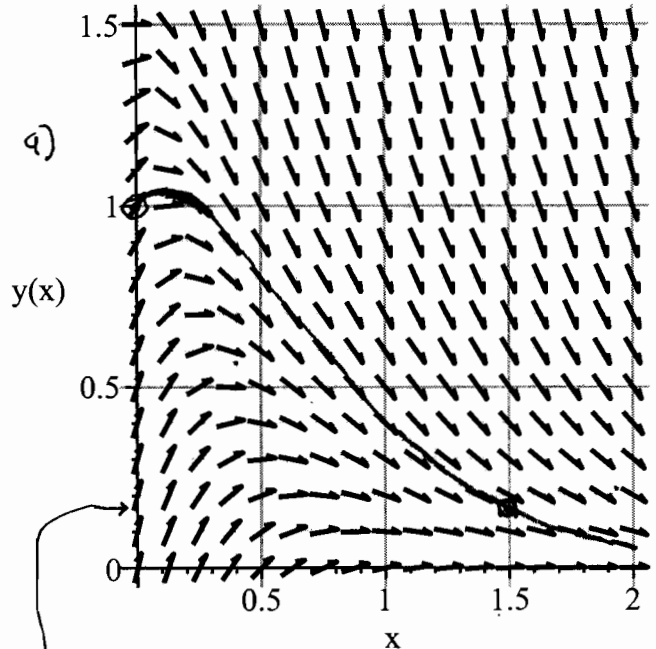
notice that the vertical grid spacing is 0.1

$y(1.5) \approx 0.175$? hard to be precise

$\dfrac{dy}{dx} \neq \dfrac{d}{dx} \longrightarrow$ only "$\dfrac{d}{dx}()$" says differentiate "()" wrt x

▶ **solution**

① b) $\left[\dfrac{dy}{dx} \boxed{+ 2} y = 3e^{-3x} \right]$ standard linear form

$\int 2\,dx = 2x \quad e \quad$ integrating factor

$\dfrac{d}{dx}(ye^{2x}) = 3e^{2x}e^{-3x} = 3e^{-x}$

$ye^{2x} = \int 3e^{-x}\,dx = -3e^{-x} + C$

$\boxed{y = e^{-2x}(-3e^{-x} + c)} \leftarrow$ product not simple here

$= -3e^{-3x} + Ce^{-2x}$ gen. soln.

c) $1 = y(0) = -3 + C \rightarrow C = 1 + 3 = 4$

$\boxed{y = -3e^{-3x} + 4e^{-2x}}$ IVP soln.

always simplify

$\dfrac{4}{e^{2x}} = 4e^{-2x}$

not simple   simple.

d) $y(1.5) = -3e^{-3(1.5)} + 4e^{-2(1.5)}$

Maple
$\approx 0.16582 \approx \boxed{0.17}$

yes, pretty close to 0.175 estimate. No extra room for triangled dot — too close to squared dot.

e) Maple's Result:
$y(x) = (-3e^{-x} + 4)e^{-2x}$ ⟩ Expand

$= \dfrac{-3}{(e^x)^3} + \dfrac{4}{(e^x)^2}$

$= -3e^{-3x} + 4e^{-2x}$ ⟩ Simplify, Simplify

same!

$\begin{pmatrix} \text{a sum of exponentials is} \\ \text{simpler than a product} \end{pmatrix}$