Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

1. $\dfrac{dy}{dx} = (x\,y)^2$

initial condition 1: $y(0) = 1$;
initial condition 2: $y(0) = 0$;
initial condition 3: $y(0) = -1$.

a) Indicate these initial data points on the graph by circled dots and roughly draw in the corresponding solution curves.

b) Find the (almost) general solution of the differential equation. What obvious solution is missing from this family?

c) Find the solution which satisfies the last initial condition.

d) Evaluate $y(1.5)$ for this solution and mark the corresponding point on the graph by a circled dot. Is this consistent with your approximate hand drawn solution? Explain.
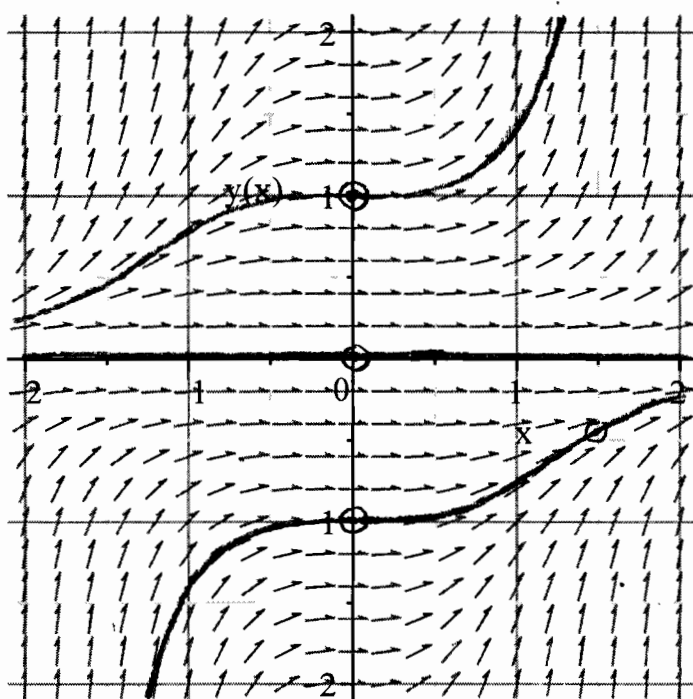
e) Check by hand that your solution to c) solves the differential equation. [Remember, backsub everywhere in the DE eliminating y, then simplify both sides independently.]

f) Enter the differential equation and the last initial condition separated by a comma in Maple. Right click and solve. Write down the form of the solution that it gives you. Does it agree with your hand solution?

▶ solution

a) See graph.

b) $\dfrac{dy}{dx} = (x\,y)^2 = x^2 y^2$ separable

$\displaystyle\int y^{-2}\,dy = \int x^2\,dx$ separate, integrate

$-y^{-1} = \dfrac{x^3}{3} + C_1$

$y^{-1} = -\dfrac{x^3}{3} + C_1$

$\boxed{y = \dfrac{1}{C_1 - \frac{x^3}{3}} = \dfrac{1}{\left(\frac{3C_1 - x^3}{3}\right)} = \dfrac{3}{3C_1 - x^3} = \dfrac{3}{C - x^3}}$ ← either acceptable, second "simpler"

c) $y(0) = -1 \leftrightarrow x = 0, y = -1$

$-1 = \dfrac{3}{C - 0^3} = \dfrac{3}{C} \rightarrow C = -3$ $\boxed{y = \dfrac{3}{-3 - x^3} = -\dfrac{3}{3 + x^3}}$

e) $\dfrac{dy}{dx} = \dfrac{d}{dx}\left(-3(3+x^3)^{-1}\right) = (-3)(-1)(3+x^3)^{-2}(0+3x^2) = \dfrac{9x^2}{(3+x^3)^2}$

↳ D.E.: $\dfrac{9x^2}{(3+x^3)^2} \overset{?}{=} x^2\left(\dfrac{-3}{3+x^3}\right)^2 = \dfrac{9x^2}{(3+x^3)^2}$ ✓

d) $y(1.5) = -\dfrac{3}{3 + (1.5)^3} \approx \boxed{-0.4706}$ [Maple]

circled dot on lowest curve at $x = 1.5$ has $y \approx -0.45$? hard to be precise, but clearly close enough given the imprecision of the hand curve drawing.

f) Maple: $\boxed{y(x) = -\dfrac{3}{3 + x^3}}$

if you simplified your answer it is the same, otherwise factoring your sign will make them agree.