MAT2705-03/04 11s Quiz 1 Print Name (Last, First) ________________ , ________________|____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

1. $2\frac{dy}{dx} + y = x, \quad soln: \; y = -2 + x + C\,e^{-\frac{1}{2}x}$

a) Verify that this y satisfies the given differential equation.

b) Find the solution which satisfies the initial condition $y(0) = 3$.

Organize your work as though you were playing professor.

2. Write a differential equation that models the situation:

"The time rate of change of a population P is proportional to the square root of the population."

Explain what sign the constant of proportionality should have and why.

[First quiz hint: Use the variable t for time and use d/dt derivative notation.]

▶ solution

① a) $y = -2 + x + Ce^{-x/2}$

$\frac{dy}{dx} = 0 + 1 + Ce^{-\frac{x}{2}}\left(-\frac{1}{2}\right) = 1 - \frac{C}{2}e^{-\frac{x}{2}}$

$2\frac{dy}{dx} + y = x$ backsub! (everywhere in DE)

$2\left(1 - \frac{C}{2}e^{-\frac{x}{2}}\right) + \left(-2 + x + Ce^{-\frac{x}{2}}\right) \overset{?}{=} x$

$\cancel{2} - \cancel{Ce^{-\frac{x}{2}}} - \cancel{2} + \cancel{Ce^{-\frac{x}{2}}} + x = x$

$x = x$ ✓

b) $y(0) = 3 \Leftrightarrow x = 0, y = 3$

$y = -2 + x + ce^{-\frac{x}{2}}$

$3 = -2 + 0 + C\underbrace{e^0}_{1}$

$\hookrightarrow C = 5$ backsub!

$\boxed{y = -2 + x + 5e^{-\frac{x}{2}}}$

② $\frac{dP}{dt} \propto \sqrt{P} \rightarrow \boxed{\frac{dP}{dt} = k\sqrt{P}} = kP^{1/2}$ (power notation better suited to Calculus rules!)

[Note $P \geq 0$, since population cannot be negative.]

In this model $\sqrt{P}$ is real only if $P \geq 0$.

In the absence of other information we expect the population to grow so $\frac{dP}{dt} > 0$ which implies $\boxed{k > 0.}$

BUT this could be a model of decreasing population (why not?), so $k < 0$ might be possible. The important thing in using a model is to make clear the assumptions of the model.

$\underbrace{\text{The time rate of change of a population } P}_{\frac{dP}{dt}}$ $\underbrace{\text{is proportional to}}_{\propto}$ $\underbrace{\text{the square root of the population}}_{\sqrt{P}}$

Note k is the sign of the rate of change of P, not of P itself!