MAT2705-03/04 11F Quiz 2 Print Name (Last, First) _____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

1. $\frac{dy}{dx} = -\frac{2}{x}y + x$, $y(1) = \frac{1}{2}$, $x \geq 0$.

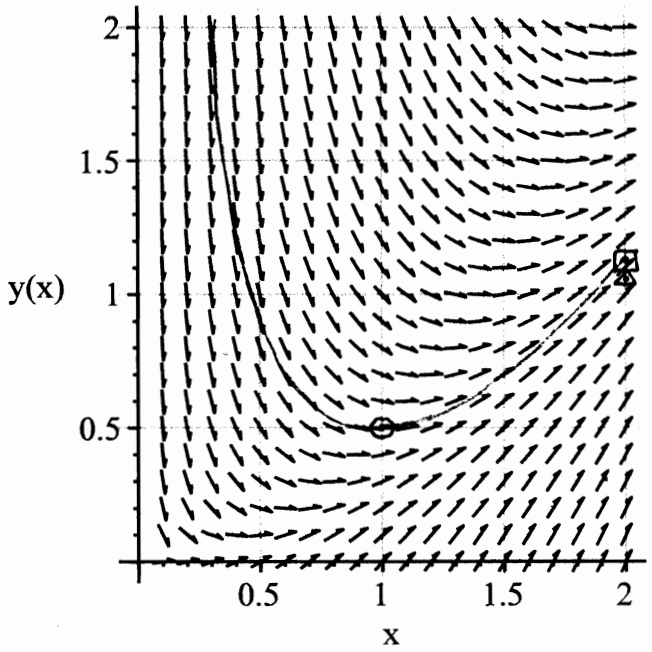
a) Hand draw in the solution of this differential equation satisfying the initial condition on the associated direction field to the right. Put a circled dot at the point corresponding to the initial condition. Put a squared dot on the curve at $x = 2$. Estimate your approximate value of $y(2)$.

b) Use the linear solution recipe to find the general solution of this differential equation. Simplify it and box it.

c) Find the solution of this differential equation which satisfies the given initial condition. Simplify it and box it.

d) Evaluate $y(2)$ numerically to 2 decimal places and mark the corresponding point on your graph with a triangled dot. Is this consistent with your part a) result? Explain.

e) Does your initial value problem solution agree with Maple (which might require Expand to multiply out, and Simplify, Simplify to simplify)? If not, can you find your mistake? If so, show the equivalence of your solution with Maple's. Did you simplify your solution as requested before comparing it to Maple's?



notice that the grid spacing is 0.1

(If you mistakenly put (½, 1) as the initial point, you actually get the same solution curve almost)

▶ solution

a) $y(1) = \frac{1}{2} \leftrightarrow x = 1, y = \frac{1}{2}$ point $(1, \frac{1}{2})$

see diagram $\boxed{y(2) \approx 1.12}$

b) standard linear form:

$x^2\left[\frac{dy}{dx} + \frac{2}{x}y = x\right]$

$e^{\int \frac{2}{x}dx} = e^{2\ln|x|} = e^{\ln|x|^2} = x^2$

$\frac{d}{dx}(yx^2) = x \cdot x^2 = x^3$

$yx^2 = \int x^3 dx = \frac{x^4}{4} + C$

$\boxed{y = x^{-2}\left(\frac{x^4}{4} + C\right) = \frac{x^2}{4} + Cx^{-2} = \frac{x^2}{4} + \frac{C}{x^2}}$

expand out — either form acceptable

c) $\frac{1}{2} = \frac{1^2}{4} + \frac{C}{1^2} = \frac{1}{4} + C \rightarrow C = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

$\boxed{y = \frac{1}{4}x^2 + \frac{1}{4x^2} = \frac{1}{4}\left(x^2 + \frac{1}{x^2}\right)}$

d) $y(2) = \frac{1}{4}\left(2^2 + \frac{1}{2^2}\right) = \frac{1}{4}\left(4 + \frac{1}{4}\right) = 1\frac{1}{16}$

$1.0625 \approx \boxed{1.06} \leftrightarrow 1.12$?

given the grid limitations, not bad

my curve rose too quickly from a horizontal tangent line at the initial point.

e) Maple returns the quotient form $y = \frac{\frac{1}{4}x^4 + \frac{1}{4}}{x^2}$ which must be "expanded" to simplify

Then it agrees with my hand result.