MAT2705-03/04 11f Quiz 1   Print Name (Last, First) ________________ , ________________|____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

1. $\frac{dy}{dx} + 2xy^2 = 0$, $soln: y = \frac{1}{C+x^2}$

a) Verify that this y satisfies the given differential equation.

b) Find the solution from this family which satisfies the initial condition $y(1) = 2$ and simplify your result.

c) Is there an obvious "singular solution" missing from this family?

Organize your work as though you were playing professor. If you have an extra moment, does your solution to b) coincide with Maple's expression?

2. Write a differential equation that models the 1-dimensional motion situation:
"The time rate of change of the velocity of a freely falling object is proportional to the square of the velocity."
[Hint: Use the variable t for time and use d/dt derivative notation.]
Can we conclude anything about the sign of the constant of proportionality? [Hint: velocity can be positive or negative, i.e., up or down.]

► solution

① a) $y = (C+x^2)^{-1}$

$\frac{dy}{dx} = -(C+x^2)^{-2}(0+2x)$

$= \frac{-2x}{(C+x^2)^2}$

$\frac{dy}{dx} + 2xy^2 = 0$

$\downarrow$

$\frac{-2x}{(C+x^2)^2} + 2x\left(\frac{1}{C+x^2}\right)^2 \stackrel{?}{=} 0$

$\frac{-2x}{(C+x^2)^2} + \frac{2x}{(C+x^2)^2} = 0$

$0 = 0$ ✓

b) $y(1) = 2 \rightarrow x=1, y=2$

$y = \frac{1}{C+x^2}$

$\downarrow$

$2 = \frac{1}{C+1}$.

$\frac{1}{2} = C+1$

$C = -\frac{1}{2}$

$y = \frac{1}{-\frac{1}{2}+x^2} = \frac{1}{x^2-\frac{1}{2}}$

$= \frac{1}{\left(\frac{2x^2-1}{2}\right)} = \frac{2}{2x^2-1}$

$\boxed{y = \frac{2}{2x^2-1}}$ ← Maple soln expression exactly

(This question does not ask about particular values of x where the expression for y has trouble.)

c) $y=0$ is an obvious soln of the DE, not in this family.

NOTE: Verifying or checking a solution does not mean rederiving it through some solution technique — it means seeing whether the solution satisfies the DE! You can misapply a soln technique or simply make mistakes in deriving the soln. Only by BACKSUBSTITUTION can you decide if it is correct.

② $\frac{dv}{dt} \propto v^2$

$\boxed{\frac{dv}{dt} = kv^2}$

without further information we cannot conclude anything about the sign of k.

(to get this right we would need an additional constant acceleration term for the gravity implied in "freely falling".)