MAT2500-03/04 10S Quiz 8 Print Name (Last, First) ________________________|__

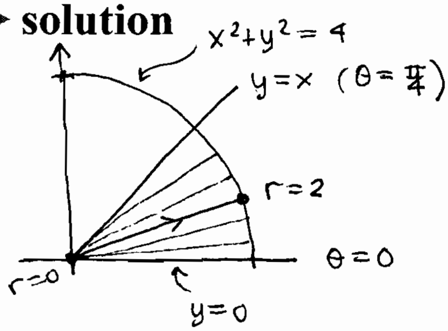
Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of each problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC).

1.b) Set up and simplify but do not evaluate the polar coordinate iterated integral of the function $f(x, y) = 2\,x\,y$ over the region of the first quadrant enclosed by the three curves $y = 0$, $y = x$, and $x^2 + y^2 = 4$ but first
a) Make a diagram of the region of integration in the xy-plane showing one typical radial cross-section with its endpoints labeled, and "shade in" the region with equally spaced radial cross-section line segments.

2.b) Use polar coordinates to find the volume of the solid region below the paraboloid $z = 18 - 2\,x^2 - 2\,y^2$ and above the xy-plane, but first
a) Make a diagram of the region of integration in the xy-plane showing one typical radial cross-section with its endpoints labeled, and "shade in" the region with equally spaced radial cross-section line segments.
Be sure to show the step-by-step evaluation process for your double integral in polar coordinates.

► **solution**

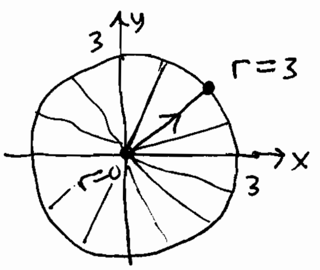
① a)



b) $\displaystyle\int_0^{\pi/4}\int_0^2 2\underbrace{(r\cos\theta)}_{x}\underbrace{(r\sin\theta)}_{y}\,\underbrace{r\,dr\,d\theta}_{dA}$

$\displaystyle = \boxed{\int_0^{\pi/4}\int_0^2 2r^3\cos\theta\sin\theta\,dr\,d\theta}$

② a) $z = 18 - 2(x^2+y^2) = 18 - 2r^2 = 0 \rightarrow r^2 = 9,\ r = 3 \rightarrow$ intersects $z=0$ plane in a circle of radius 3



b) $\displaystyle V = \underbrace{\int_0^{2\pi}\int_0^3 (18 - 2r^2)\,\underbrace{r\,dr\,d\theta}_{dA}}$ $\displaystyle = \int_0^{2\pi} \frac{81}{2}\,d\theta$

$\displaystyle \int_0^3 18r - 2r^3\,dr$ $\displaystyle = \frac{81}{2}\theta\Big|_0^{2\pi}$

$\displaystyle = 18\frac{r^2}{2} - 2\frac{r^4}{4}\Big|_{r=0}^{r=3}$ $\displaystyle = \frac{81}{2}(2\pi) = \boxed{81\pi}$

$\displaystyle = 9\cdot 9 - \frac{1}{2}81 = 81\left(1-\frac{1}{2}\right) = \frac{81}{2}$