MAT2500-03/04 10S Quiz 7 Print Name (Last, First) ____________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of each problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC).

bob makes a el cheapo 1 cubic foot terrarium with base dimensions x and y and height z in feet. The four sides are made of 1 dollar per square foot plastic, while the cheapo base is 1/4 dollar per square foot cardboard.

a) Make a generic diagram of the terrarium labeling the sides.

b) Write down the constraint (condition) on the 3 dimensions which fixes the volume as described above.

c) Write down the cost function C which is to be minimized so the cheapo terrarium is the most cheapo possible.

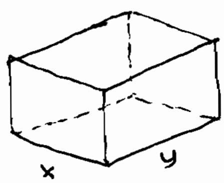
d) What is the function of 2 variables to be extremized once we eliminate one variable using the constraint?

e) Find its unique critical point (the minimum!), and the dimensions of the el cheapo terrarium which minimize its cost, and the minimal cost, and make a new sketch of this terrarium with the dimensions you found.

f) Verify that this is a local minimum of the function of part d) using the second derivative test.

▶ solution

a)



b) $V = \boxed{xyz = 1} \longrightarrow z = \frac{1}{xy}$

c) $\boxed{C = 1\cdot(\text{area sides}) + \frac{1}{4}(\text{area base}) = \frac{1}{4}xy + 2z(x+y)}$

d) solve constraint for z, eliminate z in C:

$C = \frac{1}{4}xy + 2\left(\frac{1}{xy}\right)(x+y)$

$= \boxed{\frac{1}{4}xy + 2(x^{-1}+y^{-1}) = C(x,y)}$

e) $C_x = \frac{1}{4}y - 2x^{-2} = 0 \to y = \frac{8}{x^2}$

$C_y = \frac{1}{4}x - 2y^{-2} = 0 \to x = \frac{8}{y^2} = \frac{8}{(8/x^2)^2} = \frac{8}{8^2}x^4 = \frac{x^4}{8}$

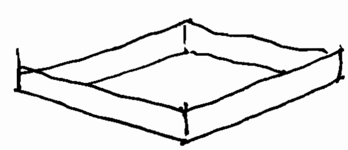
$\hookrightarrow x^4 = 8x$

$x^3 = 8$

$x = 2 \to y = \frac{8}{2^2} = 2 \to z = \frac{1}{2\cdot 2} = \frac{1}{4}$

$\boxed{\text{Base dimensions } 2\text{ ft by } 2\text{ ft}, \text{ height } 1/4 \text{ ft} = 3\text{ in}}$

Cost : $C(2,2) = \frac{1}{4}(2)(2) + 2\left(\frac{1}{2}+\frac{1}{2}\right) = 1+2 = \boxed{3 \text{ bucks}}$



height is 1/8 base dimensions

f) $C_{xx} = 4x^{-3}$ $\quad C_{xx}(2,2) = \frac{4}{8} = \frac{1}{2} > 0$

$C_{yy} = 4y^{-3}$ $\quad C_{yy}(2,2) = \frac{4}{8} = \frac{1}{2} > 0$

$C_{xy} = \frac{1}{4}$

local min?

$C_{xx}(2,2)\,C_{yy}(2,2) - C_{xy}(2,2)^2$

$= \frac{1}{2}\left(\frac{1}{2}\right) - \left(\frac{1}{4}\right)^2 = \frac{1}{4} - \frac{1}{16} > 0$

confirms local minimum in all directions.

Easy numbers, no?