

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC).

1. $f(x, y) = x \cos(y) - \sin(xy)$

a) Evaluate f and the first partial derivatives of f at $(x, y) = (2, \pi)$ using proper notation for all derivatives evaluated in the process.

b) What can you conclude about how f is increasing or decreasing at $(2, \pi)$ as you increase x and y respectively?

T / v	5	10	15
5	4	3	2
0	-2	-3	-4
-5	-7	-9	-11

2. a) The table gives values of the wind chill temperature $W = f(T, v)$ as a function of the actual temperature T in degrees Celsius and velocity v in kph. What is the practical interpretation of the value $f(0, 10)$?

b) Estimate the value of $\frac{\partial f}{\partial v}(0, 10)$ by averaging the left and right average rates of change at the point $(0, 10)$, just like you did when you did this homework problem. :-). What are its units? Can you interpret what this value means in practical terms?

► solution

1) a) $f(x, y) = x \cos y - \sin xy$

$$f_x(x, y) = \frac{\partial}{\partial x} (x \cos y - \sin xy)$$

$$= 1 \cos y - \cos xy \frac{\partial}{\partial x} (xy)$$

$$= \boxed{\cos y - y \cos xy} = \frac{\partial f}{\partial x}(x, y)$$

$$f_y(x, y) = \frac{\partial}{\partial y} (x \cos y - \sin xy)$$

$$= x(-\sin y) - \cos xy \frac{\partial}{\partial y} (xy)$$

$$= \boxed{-x \sin y - x \cos xy} = \frac{\partial f}{\partial y}(x, y)$$

$$f(2, \pi) = 2 \cos \pi - \sin 2\pi = -2 - 0 = \boxed{-2}$$

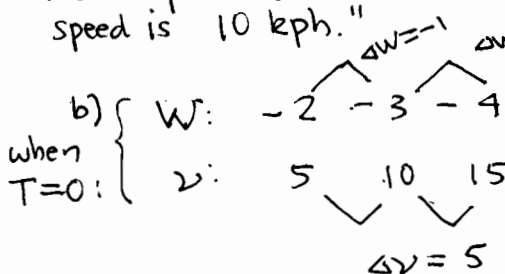
$$f_x(2, \pi) = \cos \pi - \pi \cos 2\pi = \boxed{-1 - \pi} < 0$$

$$f_y(2, \pi) = -2 \sin \pi - 2 \cos 2\pi = 0 - 2 = \boxed{-2} < 0$$

b) Since both derivatives are negative, f is decreasing as you increase either x or y .

2) a) $f(0, 10) = -3$ means

" -3°C is the effective temperature we feel when the real temperature is 0°C and the wind speed is 10 kph."



$$\frac{\Delta W}{\Delta v} \Big|_L = -\frac{1}{5} \quad \frac{\Delta W}{\Delta v} \Big|_R = -\frac{1}{5}$$

$$\frac{\partial W}{\partial v}(0, 10) = \frac{1}{2} \left(-\frac{1}{5} - \frac{1}{5} \right) = \boxed{-\frac{1}{5}}$$

units are $^\circ\text{C} / \text{kph}$.

" If we increase the wind speed by 1 kph at 10 kph (ie, to 11 kph) when the temperature is 0°C , the windchill should decrease by 0.2° "