MAT2500-03/04 10S Quiz 3 Print Name (Last, First) ________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

Given the vector-valued function $\vec{r}(t) = \langle 4\sqrt{t}, t, t^2 \rangle$ for the domain $t \geq 0$ (no credit for unidentified expressions):

a) Evaluate $\vec{r}\,'(t)$, $\vec{r}\,''(t)$, $|\vec{r}\,'(t)|$, $\vec{T}(t)$ and remember to simplify your results.

b) Evaluate $\vec{r}(1)$, $\vec{r}\,'(1)$, $\vec{r}\,''(1)$, $\vec{T}(1)$ and remember to simplify your results.

c) Evaluate the exact angle θ in radians between $\vec{r}\,'(1)$ and $\vec{r}\,''(1)$ and a single decimal place approximation in degrees.

d) Evaluate the vector $\vec{w}$ which is the vector projection of $\vec{r}\,''(1)$ orthogonal (perpendicular!) to $\vec{r}\,'(1)$.

▶ solution

a)
$$\boxed{\begin{aligned}\vec{r} &= \langle 4t^{1/2}, t, t^2\rangle \\ \vec{r}\,' &= \langle 2t^{-1/2}, 1, 2t\rangle \\ \vec{r}\,'' &= \langle -t^{-3/2}, 0, 2\rangle\end{aligned}}$$

$$|\vec{r}\,'| = \sqrt{\left(\frac{2}{t^{1/2}}\right)^2 + 1^2 + (2t)^2} = \sqrt{\frac{4}{t} + 1 + 4t^2} = \boxed{\sqrt{\frac{4+t+4t^3}{t}}}$$

$$\hat{T} = \frac{\vec{r}\,'}{|\vec{r}\,'|} = \frac{t^{1/2}}{\sqrt{4+t+4t^3}}\left\langle \frac{2}{t^{1/2}}, 1, 2t\right\rangle = \boxed{\frac{\langle 2, t^{1/2}, 2t^{3/2}\rangle}{\sqrt{4+t+4t^3}}}$$

simplest form!

b)
$$\boxed{\begin{aligned}\vec{r}(1) &= \langle 4, 1, 1\rangle \\ \vec{r}\,'(1) &= \langle 2, 1, 2\rangle \\ \vec{r}\,''(1) &= \langle -1, 0, 2\rangle \\ \hat{T}(1) &= \frac{\langle 2, 1, 2\rangle}{3}\end{aligned}}$$

$\sqrt{9}$!

c)
$$\widehat{\vec{r}\,''(1)} = \frac{\langle -1, 0, 2\rangle}{\sqrt{5}}$$

$$\cos\theta = \widehat{\vec{r}\,''(1)} \cdot \hat{T}(1) = \frac{\langle -1, 0, 2\rangle}{\sqrt{5}} \cdot \frac{\langle 2, 1, 2\rangle}{3} = \frac{-2+4}{3\sqrt{5}} = \frac{2}{3\sqrt{5}}$$

$$\boxed{\theta = \arccos\frac{2}{3\sqrt{5}} \approx 72.7°}$$

d)
$$\underbrace{\vec{r}\,''(1)}_{\vec{a}} \cdot \hat{T}(1) = \langle -1, 0, 2\rangle \cdot \frac{\langle 2, 1, 2\rangle}{3} = \frac{-2+4}{3} = \frac{2}{3} \quad \text{scalar projection}$$

$$\vec{a}_{\parallel} = \frac{2}{3}\hat{T}(1) = \frac{2}{3}\frac{\langle 2, 1, 2\rangle}{3} = \frac{2}{9}\langle 2, 1, 2\rangle = \left\langle \frac{4}{9}, \frac{2}{9}, \frac{4}{9}\right\rangle$$

$$\vec{a}_{\perp} = \vec{a} - \vec{a}_{\parallel} = \langle -1, 0, 2\rangle - \left\langle \frac{4}{9}, \frac{2}{9}, \frac{4}{9}\right\rangle = \left\langle \frac{-9-4}{9}, -\frac{2}{9}, \frac{18-4}{9}\right\rangle$$

$$= \boxed{\left\langle -\frac{13}{9}, -\frac{2}{9}, \frac{14}{9}\right\rangle = \vec{w}}$$