MAT2500-03/04 10S Test 2

① a) $f(xyz) = \frac{x+y}{z} = (x+y)z^{-1}$

$\vec{\nabla} f(xyz) = \langle \frac{1}{z}, \frac{1}{z}, -\frac{(x+y)}{z^2} \rangle$

$f(1,1,-1) = \frac{1+1}{-1} = -2$

$\vec{\nabla} f(1,1,-1) = \langle -1,-1,-2 \rangle$
$= -\langle 1,1,2 \rangle$

$\boxed{\vec{n} = \langle 1,1,2 \rangle}$

$\vec{n} \cdot (\vec{r} - \vec{r_0}) = 0$

$\langle 1,1,2 \rangle \cdot (\vec{r} - \langle 1,1,-1 \rangle) = 0$

$1(x-1) + 1(y-1) + 2(z+1) = 0$

$x + y + 2z - 1 - 1 + 2 = 0$

$\boxed{x + y + 2z = 0}$

level surface: $\boxed{\frac{x+y}{z} = -2}$

b) $D_{\hat{u}} f(1,1,-1) = \hat{u} \cdot \vec{\nabla} f(1,1,-1)$
$= \frac{\langle 1,2,2 \rangle}{3} \cdot (-\langle 1,1,2 \rangle)$
$= -\frac{(1+2+4)}{3} = \boxed{-\frac{7}{3}}$

c) $\vec{v} = \widehat{\vec{\nabla} f(1,1,-1)} = \boxed{\frac{-\langle 1,1,2 \rangle}{\sqrt{6}}}$

d) $L(x,y,z) = f(1,1,-1) + f_x(1,1,-1)(x-1)$
$+ f_y(1,1,-1)(y-1) + f_z(1,1,-1)(z+1)$

$\boxed{L(xyz) = -2 - 1(x-1) - 1(y-1) - 2(z+1)}$

$L(1.01, 0.98, -1.02)$
$= -2 - 1(\underbrace{1.01-1}_{.01}) - (\underbrace{0.98-1}_{-0.02}) - 2(\underbrace{-1.02+1}_{-0.02})$
$= -2 - 0.01 + 0.02 + 0.04$
$= -2 \underbrace{+0.05}_{\text{small change!}} = \boxed{-1.95}$

$\approx f(1.01, 0.98, -1.02)$

② a) $z = e^{x-y}\cos(x+y) = f(x,y)$

$f_x = e^{x-y}\cos(x+y) - e^{x-y}\sin(x+y)$

$f_y = -e^{x-y}\cos(x+y) - e^{x-y}\sin(x+y)$

$f(0,0) = e^0 \cos 0 = 1$

$f_x(0,0) = e^0(\cos 0 - \sin 0) = 1$

② a) continued

$f_y(0,0) = -e^0(\cos 0 + \sin 0) = -1$

$L(x,y) = f(0,0) + f_x(0,0)(x-0) + f_y(0,0)(y-0)$
$= 1 + 1(x-0) + -1(y-0) = 1 + x - y$

tangent plane $z = L(x,y) = 1 + x - y$

$\boxed{-x + y + z = 1}$

OR: $F(x,y,z) = z - e^{x-y}\cos(x+y)$

$\vec{\nabla} F(x,y,z) = \langle -f_x(x,y), -f_y(x,y), 1 \rangle$

$\vec{\nabla} F(0,0,1) = \langle -1, 1, 1 \rangle = \vec{n}$

$\vec{n} \cdot (\vec{r} - \vec{r_0}) = 0 \qquad \vec{r_0} = \langle 0,0,1 \rangle$

$\langle -1,1,1 \rangle \cdot \langle x, y, z-1 \rangle = 0$

$-x + y + z - 1 = 0$, $\boxed{-x + y + z = 1}$

b) coeffs of variables: $\boxed{\vec{n} = \langle -1,1,1 \rangle}$
(or $\langle 1,-1,-1 \rangle$).

c) $\vec{r} = \vec{r_0} + t\vec{n} = \langle 0,0,1 \rangle + t\langle -1,1,1 \rangle$
$= \boxed{\langle -t, t, 1+t \rangle = \langle x,y,z \rangle}$

③ a) $f(x,y) = xye^{-x^2-y^2}$

$f_x = ye^{-x^2-y^2} + xye^{-x^2-y^2}(-2x)$
$= y(1-2x^2)e^{-x^2-y^2} = 0 \rightarrow y = 0$ or $x = \frac{1}{\sqrt{2}}$

$f_y = xe^{-x^2-y^2} + xye^{-x^2-y^2}(-2y)$
$= x(1-2y^2)e^{-x^2-y^2} = 0 \rightarrow x = 0$ or $y = \frac{1}{\sqrt{2}}$

$(0,0)\ (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$ only combinations which make both f_x and f_y zero at same time!

$f_{xx} = y(-4x)e^{-x^2-y^2} + y(1-2x^2)e^{-x^2-y^2}(-2x)$
$= -2xy(2+1-2x^2)e^{-x^2-y^2} = -2xy(3-2x^2)e^{-x^2-y^2}$

$f_{yy} = x(-4y)e^{\cdots} + x(1-2y^2)e^{\cdots}(-2y) = -2xy(2$
$= -2xy(2+1-2y^2)e^{\cdots} = -2xy(3-2y^2)e^{-x^2-y^2}$

$f_{xy} = (1-2x^2)e^{\cdots} + y(1-2x^2)e^{-x^2-y^2}(-2y)$
$= (1-2x^2)(1-2y^2)e^{-x^2-y^2}$

	$(0,0)$	$(1/\sqrt{2}, 1/\sqrt{2})$
f_{xx}	0 inconclusive	$-\frac{3}{2}(2)e^{-1} < 0 \Rightarrow$ local max
f_{yy}	0 inconclusive	$-\frac{3}{2}(2)e^{-1} < 0 \Rightarrow$ local max
f_{xy}	1	0
$f_{xx}f_{yy} - f_{xy}^2$	$-1 < 0$ saddle	$+4e^{-2} > 0$ confirms local max