MAT2500-03/04 10S Test 1 Answers (1)

a) $\vec{r}(t) = \langle t, t^2, 1-t^4\rangle$

$\vec{r}\,'(t) = \langle 1, 2t, -4t^3\rangle$

$\vec{r}\,''(t) = \langle 0, 2, -12t^2\rangle$ oops! correction

$= 2\langle 0, 1, -6t^2\rangle$

$|\vec{r}\,'(t)| = \sqrt{1+(2t)^2+(-4t^3)^2}$

$= \sqrt{1+4t^2+16t^6}$

$\hat{T}(t) = \dfrac{\vec{r}\,'(t)}{|\vec{r}\,'(t)|} = \dfrac{\langle 1, 2t, -4t^3\rangle}{\sqrt{1+4t^2+16t^3}}$

$|\vec{r}\,''(t)| = 2\sqrt{0+1+(-6t^2)^2}$

$= 2\sqrt{1+36t^4}$

$\vec{r}(-\tfrac12) = \langle -\tfrac12, \tfrac14, 1-\tfrac1{16}\rangle = \langle -\tfrac12, \tfrac14, \tfrac{15}{16}\rangle$

$\vec{r}\,'(-\tfrac12) = \langle 1, -1, -4(-\tfrac18)\rangle = \langle 1, -1, \tfrac12\rangle$

$\vec{r}\,''(-\tfrac12) = \langle 0, 2, -12(\tfrac14)\rangle = \langle 0, 2, -3\rangle$

$|\vec{r}\,'(-\tfrac12)| = \sqrt{1+1+\tfrac14} = \sqrt{\tfrac94} = \tfrac32$

$\hat{T}(-\tfrac12) = \dfrac{\langle 1, -1, \tfrac12\rangle}{3/2} = \dfrac{\langle 2, -2, 1\rangle}{3}$

$|\vec{r}\,''(-\tfrac12)| = 2\sqrt{1+36(\tfrac1{16})} = \sqrt{4+9} = \sqrt{13}$

b) $\vec{r}\,'(t)\times\vec{r}\,''(t) = \begin{vmatrix} i & j & k \\ 1 & 2t & -4t^3 \\ 0 & 2 & -12t^2\end{vmatrix}$

$= \langle -24t^3+8t^3, 0+12t^2, 2-0\rangle$

$= \langle -16t^3, 12t^2, 2\rangle$

$= 2\langle -8t^3, 6t^2, 1\rangle$

$\boxed{\hat{B}(t) = \dfrac{\langle -8t^3, 6t^2, 1\rangle}{\sqrt{1+36t^4+64t^6}}}$

c) $\vec{r}(-\tfrac12) = \langle -\tfrac12, \tfrac14, \tfrac{15}{16}\rangle = \vec{r}_0$

$\langle -8(-\tfrac12)^3, 6(-\tfrac12)^2, 1\rangle$

$= \langle 1, \tfrac32, 1\rangle = \dfrac{\langle 2, 3, 2\rangle}{2}$

$\vec{n} = \langle 2, 3, 2\rangle$ is $\perp$ this plane

$\vec{n}\cdot(\vec{r}-\vec{r}_0) = 0$

$\langle 2, 3, 2\rangle\cdot\langle x+\tfrac12, y-\tfrac14, z-\tfrac{15}{16}\rangle = 0$

$2(x+\tfrac12)+3(y-\tfrac14)+2(z-\tfrac{15}{16}) = 0$

$2x+3y+2z+\underbrace{1-\tfrac34-\tfrac{15}{8}}_{-\frac{13}{8}} = 0$

$\boxed{2x+3y+2z = \tfrac{13}{8}}$

d) $\vec{b} = \vec{r}\,'(-\tfrac12) = \langle 1, -1, \tfrac12\rangle$ "along tangent vector"

$\vec{r} = \vec{r}_0 + t\vec{b} = \langle -\tfrac12, \tfrac14, \tfrac{15}{16}\rangle + t\langle 1, -1, \tfrac12\rangle$

$\boxed{\langle x, y, z\rangle = \langle -\tfrac12+t, \tfrac14-t, \tfrac{15}{16}+\tfrac12 t\rangle}$

e) set $z = \tfrac{15}{16}+\tfrac12 t = 1 \to \tfrac12 t = 1-\tfrac{15}{16} = \tfrac1{16}$

$\to t = \tfrac18 \to x = -\tfrac12+\tfrac18 = -\tfrac38$, $y = \tfrac14 - t = \tfrac18$

so $\boxed{\langle x, y, z\rangle = \langle -\tfrac38, \tfrac18, 1\rangle}$

f) $L = \int_0^1 |\vec{r}\,'(t)|\,dt = \int_0^1 \sqrt{1+4t^2+16t^6}\,dt$

$\overset{\text{maple}}{\approx} \boxed{1.911123500}$

$\vec{r}(0) = \langle 0, 0, 1\rangle$

$\vec{r}(1) = \langle 1, 1, 0\rangle$

$\vec{r}(1)-\vec{r}(0) = \langle 1, 1, 0\rangle - \langle 0, 0, 1\rangle = \langle 1, 1, -1\rangle$

$L_{SL} = |\vec{r}(1)-\vec{r}(0)| = \sqrt{1+1+1} = \sqrt3 \approx \boxed{1.732}$

$L/L_{SL} \approx \boxed{1.103}$, $\boxed{\text{L is about 10\% longer}}$

yes the curve should be a bit longer than the straight line, given the diagram.

g) $\vec{a} = \langle 0, 2, -3\rangle$, $\hat{T}(-\tfrac12) = \dfrac{\langle 2, -2, 1\rangle}{3}$

$a_T = \vec{a}\cdot\hat{T} = \langle 0, 2, -3\rangle\cdot\dfrac{\langle 2, -2, 1\rangle}{3}$

$= \dfrac{-4-3}{3} = \boxed{-\tfrac73}$

$\hat{T}\times\vec{a} = \tfrac13\langle 2, -2, 1\rangle\times\langle 0, 2, -3\rangle$

$= \tfrac13\begin{vmatrix} i & j & k \\ 2 & -2 & 1 \\ 0 & 2 & -3\end{vmatrix} = \tfrac13\langle 6-2, 0+6, 4-0\rangle$

$= \tfrac13\langle 4, 6, 4\rangle = \tfrac23\langle 2, 3, 2\rangle$

$a_N = |\hat{T}\times\vec{a}| = \tfrac23\sqrt{4+9+4} = \tfrac23\sqrt{17}$

$|\vec{a}|^2 = 4+9 = 13$

$a_T^2+a_N^2 = (-\tfrac73)^2 + (\tfrac23\sqrt{17})^2 = \tfrac{49}{9}+\tfrac{4\cdot17}{9}$

$= \tfrac{49+68}{9} = 13$ ✓ (no right answer to lucky number)