MAT2705-02/06 10F Quiz 3   Print Name (Last, First) ________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

1. $2\frac{dy}{dx} + \frac{1}{x+1} y - 3 = 0\,, y(0) = 3$, $x \ge 0$.

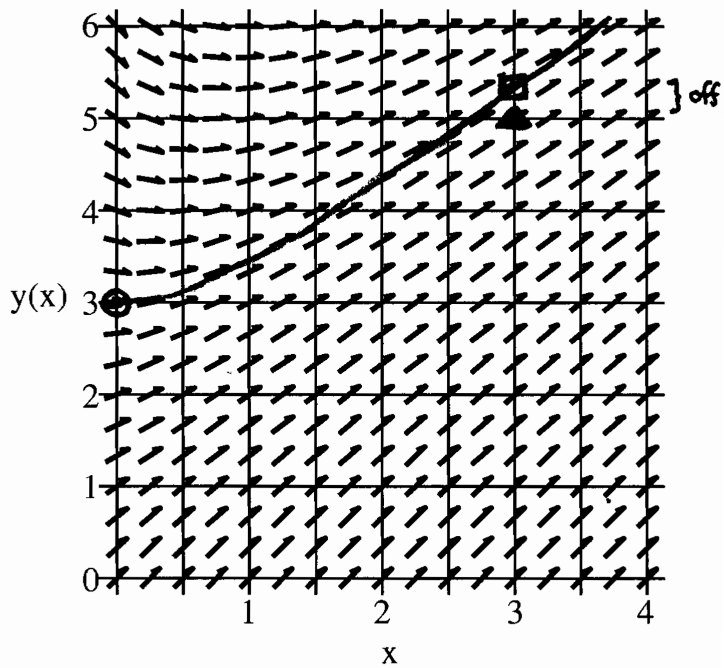
a) Hand draw in the solution of this differential equation satisfying the initial condition on the associated direction field to the right. Put a circled dot at the point corresponding to the initial condition. Put a squared dot on the curve at $x = 3$. Estimate your approximate value of $y(3)$.

b) Use the linear solution recipe to find the general solution of this differential equation. Simplify it and box it.

c) Find the solution of this differential equation which satisfies the given initial condition. Simplify it and box it.

d) Evaluate y(3) numerically to 2 decimal places and mark the corresponding point on your graph with a triangled dot. Is this consistent with your part a) result? Explain.

e) Does your initial value problem solution agree with Maple? If not, can you find your mistake?



► solution

a) Looks like $\boxed{5\tfrac{1}{3} \approx y(3)}$

b) standard form: $\frac{dy}{dx} + \underbrace{\frac{1}{2(x+1)}} y = \frac{3}{2} \rightarrow \frac{d}{dx}\left(y\,(x+1)^{1/2}\right) = \frac{3}{2}(x+1)^{1/2}$

$e^{\frac{1}{2}\int \frac{dx}{x+1}} = e^{\frac{1}{2}\ln|x+1|} = |x+1|^{1/2}$
$= (x+1)^{1/2}$ since $x \ge 0$

$y\,(x+1)^{1/2} = \frac{3}{2}\int (x+1)^{1/2}\,dx = \cancel{\frac{3}{2}}\,\frac{(x+1)^{3/2}}{\cancel{3/2}} + C$

$\boxed{y = \frac{(x+1)^{3/2} + C}{(x+1)^{1/2}} = x+1 + \frac{C}{(x+1)^{1/2}}}$

c) $3 = y(0) = 1 + C \rightarrow C = 2$, $\boxed{y = x+1 + \frac{2}{(x+1)^{1/2}}}$

d) $y(3) = 3+1 + \frac{2}{(3+1)^{1/2}} = 4 + \frac{2}{2} = \boxed{5.00} \leftrightarrow 5.33$ close but disappointingly not as close as I would like

e) $> y' + \frac{1}{2(x+1)} y = 3\,, y(0) = 3 \xrightarrow[\text{right click}]{\text{Maple}} y(x) = x+1+\frac{2}{\sqrt{x+1}}$ agrees with expanded expression (divided thru)