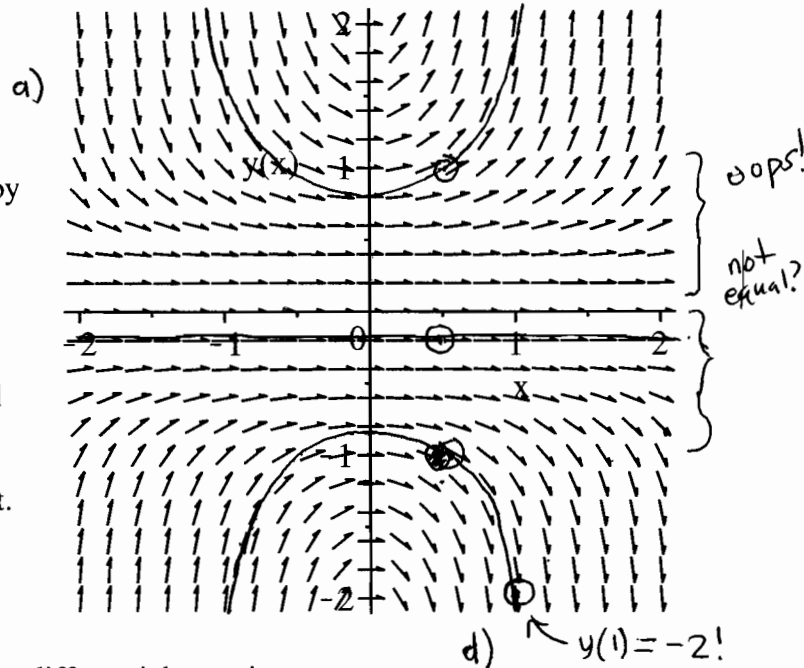


Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. **BOX** final short answers. **LABEL** parts of problem. Keep answers **EXACT** (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

1.  $\frac{dy}{dx} = xy^3$

- initial condition 1:  $y(1/2) = 1$ ;
- initial condition 2:  $y(1/2) = 0$ ;
- initial condition 3:  $y(1/2) = -1$ .

- a) Indicate these initial data points on the graph by circled dots and roughly draw in the corresponding solution curves.
- b) Find the (almost) general solution of the differential equation. What obvious solution is missing from this family?
- c) Find the solution which satisfies the last initial condition.
- d) Evaluate  $y(1)$  for this solution and mark the corresponding point on the graph by a circled dot. Is this consistent with your approximate hand drawn solution? Explain.



- e) Check by hand that your solution to c) solves the differential equation.
- f) Enter the differential equation and the last initial condition separated by a comma in Maple. Right click and solve. Write down the form of the solution that it gives you. Now right click and choose Simplify, simplify. Does it agree with your hand solution? Can you simplify Maple's solution by hand to get the same result? If so, show the steps.

► **solution**

b)  $\frac{dy}{dx} = xy^3$

$\int y^{-3} dy = \int x dx$

$\frac{y^{-2}}{-2} = \frac{x^2}{2} + C_1$

$\frac{1}{y^2} = -x^2 - \frac{2C_1}{1} = C - x^2$

$y^2 = \frac{1}{C - x^2}$

$y = \pm \frac{1}{\sqrt{C - x^2}}$

The soln  $y = 0$  is missing.

c)  $y(1/2) = -1 \rightarrow x = 1/2, y = -1$

$-1 = -\frac{1}{\sqrt{C - (1/2)^2}}$

$1 = \frac{1}{C - 1/4}$

$C - 1/4 = 1$

$C = 5/4$

(or  $\frac{1}{(-1)^2} = C - (1/2)^2$ )

$C = 1 + 1/4 = 5/4$

e)  $y = \frac{-2}{\sqrt{5-4x^2}}$

$= -2(5-4x^2)^{-1/2}$

$\frac{dy}{dx} = -2(-1/2)(5-4x^2)^{-3/2}(0-8x)$

$= \frac{-8x}{(5-4x^2)^{3/2}}$  backsub into D

$\frac{-8x}{(5-4x^2)^{3/2}} = x \left( \frac{-2}{\sqrt{5-4x^2}} \right)^3$

$= \frac{-8x}{(5-4x^2)^{3/2}} \checkmark$

f) Maple gives this, simplifies to this.

$y = -\frac{1}{\sqrt{5/4 - x^2}}$

$= -\frac{1}{\frac{\sqrt{5-4x^2}}{2}} = \frac{-2}{\sqrt{5-4x^2}}$

d)  $y(1) = -\frac{1}{\sqrt{5/4 - 1}} = -\frac{1}{\sqrt{1/4}} = -2$

(possible simplification)