MAT2705-02/06 09F Quiz 1 Print Name (Last, First) _____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

1. $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 0$, *soln*: $y(x) = 2e^{-x}$

a) Verify that $y(x)$ satisfies the given differential equation.

b) Determine all values of the constant k so that $y(x) = e^{kx}$ satisfies the differential equation; state the corresponding solutions $y(x)$.

Organize your work as though you were playing professor.

2. Write a differential equation that models the situation:
"A wet towel is hung out to dry in the sun. Assume that the rate at which the amount of moisture decreases is proportional to the amount remaining."
[First quiz hint: let a be the amount of moisture at time t. Use d/d derivative notation.] When is drying the fastest? At the beginning or towards the end? Explain. Make sure the sign of the rate of change in your DE is clear from your remarks.

▶ solution

backsub

① a) $y = 2e^{-x}$, $y' = -2e^{-x}$, $y'' = 2e^{-x}$ → $y'' + 3y' + 2y = 0$ → $2e^{-x} + 3(-2e^{-x}) + 2(2e^{-x}) = 0$

$(2 - 6 + 4)e^{-x} = 0$

$0 = 0$ ✓

b) $y = e^{kx}$, $y' = ke^{kx}$, $y'' = k^2e^{kx}$ → $y'' + 3y' + 2y = 0$ →

$k^2e^{kx} + 3ke^{kx} + 2e^{kx} = 0$

$(k^2 + 3k + 2)e^{kx} = 0$

$= (k+1)(k+2) = 0$

↳ $k = -1, -2$

$y = e^{-x}, e^{-2x}$

Slightly ambiguous: a = amount remaining in towel

② $-\frac{da}{dt}$ oc a → $-\frac{da}{dt} = ka$, $k > 0$ for positive rate of decrease

$-\frac{da}{dt}$: rate at which amount a decreases; a: amount remaining

$\frac{da}{dt} = -ka$ or $= ka$ — $k < 0$

$a \neq$ amount evaporated off, since this increases, not decreases.
I copied this right out of a textbook – in life you have to be able to reason out a statement that is not perfectly clear.

Drying is fastest at the beginning when the rate of change is greatest in absolute value