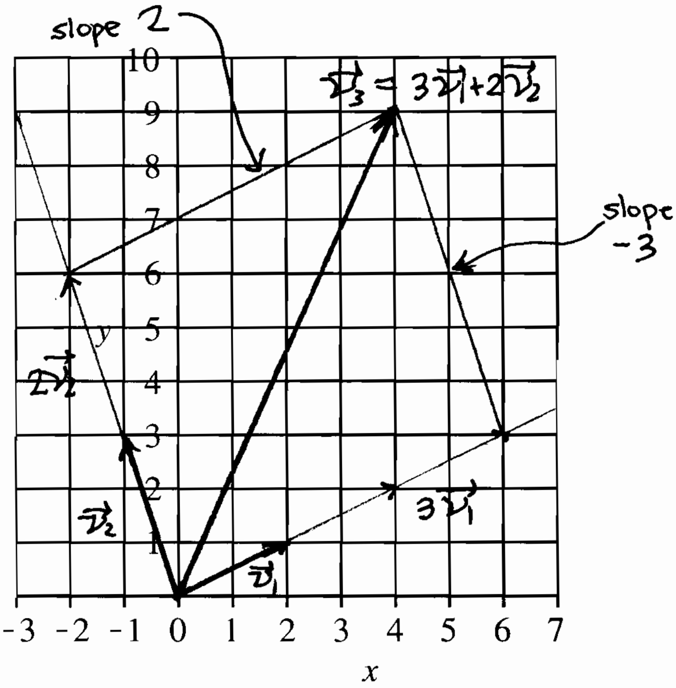
MAT 2705/02-06 10F TEST2 Answers

1 a)



From tip of $\vec{v}_3$ draw lines parallel to $\vec{v}_1, \vec{v}_2$ until they hit the lines extending $\vec{v}_1$ and $\vec{v}_2$.

Easily $(y_1, y_2) = (3, 2)$.

b) $\begin{bmatrix}4\\9\end{bmatrix} = y_1\begin{bmatrix}2\\1\end{bmatrix} + y_2\begin{bmatrix}-1\\3\end{bmatrix} = \begin{bmatrix}2 & -1\\1 & 3\end{bmatrix}\begin{bmatrix}y_1\\y_2\end{bmatrix}$

$\begin{pmatrix}y_1\\y_2\end{pmatrix} = \begin{bmatrix}2 & -1\\1 & 3\end{bmatrix}^{-1}\begin{bmatrix}4\\9\end{bmatrix} = \frac{1}{7}\begin{bmatrix}3 & 1\\-1 & 2\end{bmatrix}\begin{bmatrix}4\\9\end{bmatrix}$

$= \frac{1}{7}\begin{bmatrix}12+9\\-4+18\end{bmatrix} = \frac{1}{7}\begin{bmatrix}21\\14\end{bmatrix} = \begin{bmatrix}3\\2\end{bmatrix}$ ✓

c) check $3\begin{bmatrix}2\\1\end{bmatrix} + 2\begin{bmatrix}-1\\3\end{bmatrix} = \begin{bmatrix}6\\3\end{bmatrix} + \begin{bmatrix}-2\\6\end{bmatrix} = \begin{bmatrix}4\\9\end{bmatrix}$ ✓

d) yes, thanks bob for making simple integer coordinates.

2 a)

$$\xrightarrow{\text{start}} \xrightarrow{\text{aug}} \begin{bmatrix}3 & 1 & -1 & 9 & 0\\-4 & 2 & 8 & -2 & 0\\5 & 0 & -5 & 10 & 0\end{bmatrix} \xrightarrow{\text{RREF}} \begin{array}{c}\begin{array}{cccc}L & L & F & F\\x_1 & x_2 & x_3 & x_4\end{array}\\\begin{bmatrix}1 & 0 & -1 & 2 & 0\\0 & 1 & 2 & 3 & 0\\0 & 0 & 0 & 0 & 0\end{bmatrix}\end{array}$$

$\left.\begin{array}{l}x_3 = t_1\\x_4 = t_2\end{array}\right\} \to \begin{array}{l}x_1 - x_3 + 2x_4 = 0\\x_2 + 2x_3 + 3x_4 = 0\end{array} \to \begin{array}{l}x_1 = t_1 - 2t_2\\x_2 = -2t_1 - 3t_2\end{array}$

$\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} = \begin{bmatrix}t_1 - 2t_2\\-2t_1 - 3t_2\\t_1\\t_2\end{bmatrix}$ b) $= t_1\underbrace{\begin{bmatrix}1\\-2\\1\\0\end{bmatrix}}_{=\vec{u}_1} + t_2\underbrace{\begin{bmatrix}-2\\-3\\0\\1\end{bmatrix}}_{=\vec{u}_2}$

$\{\vec{u}_1, \vec{u}_2\}$ is a basis of the soln space

c) $\vec{v}_1 - 2\vec{v}_2 + \vec{v}_3 = \vec{0}$, $-2\vec{v}_1 - 3\vec{v}_2 + \vec{v}_4 = \vec{0}$

(coefficients are components of $\vec{u}_1, \vec{u}_2$)

d) There are only 2 independent vectors in this set since $\vec{v}_3$ & $\vec{v}_4$ can be expressed in terms of $\vec{v}_1, \vec{v}_2$ so the span of the whole set is a plane through the origin of $\mathbb{R}^3$.

$A\vec{x} = \vec{0}$: $\begin{bmatrix}3 & 1 & -1 & -9\\-4 & 2 & 8 & -2\\5 & 0 & -5 & 10\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$

3 a) $x_1\vec{v}_1 + x_2\vec{v}_2 + x_3\vec{v}_3 = \vec{v}_4$?

$\begin{bmatrix}7 & 2 & 2\\5 & 4 & 1\\3 & 6 & 3\\1 & 8 & 4\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}9\\3\\3\\1\end{bmatrix} \longrightarrow$ aug

$$\begin{bmatrix}7 & 2 & 2 & 9\\5 & 4 & 1 & 3\\3 & 6 & 3 & 3\\1 & 8 & 4 & 1\end{bmatrix} \xrightarrow{\text{RRED}} \begin{array}{c}\begin{array}{ccc}L & L & L\\x_1 & x_2 & x_3\end{array}\\\begin{bmatrix}1 & 0 & 0 & 1\\0 & 1 & 0 & -1\\0 & 0 & 1 & 2\\0 & 0 & 0 & 0\end{bmatrix}\end{array}$$

soln: $\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}1\\-1\\2\end{bmatrix}$ yes $\vec{v}_4$ lies in span $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$

$\vec{v}_4 = \vec{v}_1 - \vec{v}_2 + 2\vec{v}_3$

b) only lowerright corner entry changes

$$\begin{bmatrix}7 & 2 & 2 & 9\\5 & 4 & 1 & 3\\3 & 6 & 3 & 3\\1 & 8 & 4 & 3\end{bmatrix} \xrightarrow{\text{RRED}} \begin{array}{c}\begin{array}{ccc}x_1 & x_2 & x_3\end{array}\\\begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{bmatrix}\end{array}$$

$0 \neq 1$ inconsistent system

no soln.

no, $\vec{v}_4$ does not lie in span $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$

c) Since no free variables in homogeneous soln, $\vec{x} = \vec{0}$ is only homogeneous solution, so no linear relationships exist among these 3 vectors.