MAT2500-03/04 09S Quiz 5   Print Name (Last, First) ______________________________

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC).

1. $f(x, y, z) = 5\,x\,z - x^2 y^3 z^4$

a) Evaluate f and the first partial derivatives of f at $(x, y, z) = (-1, 1, 1)$ using proper notation for all derivatives evaluated in the process.

b) Evaluate the linear approximation function $L(x, y, z)$ to f at $(-1, 1, 1)$ and use it to approximate $f(-0.99, 0.98, 1.01)$

c) Evaluate the third partial derivative function $f_{xyz} = \dfrac{\partial^3 f}{\partial z\, \partial y\, \partial x}$ and its value at $(-1, 1, 1)$.

d) For the level surface $f(x, y, z) = -6$ passing through the point $(-1, 1, 1)$, use implicit differentiation to evaluate $\dfrac{\partial z}{\partial x}$ at this point.

e) **Optional.** The tangent plane to this surface at this point has the equation $L(x, y, z) = -6$. Solve this for z, and evaluate $\dfrac{\partial z}{\partial x}$. Does this agree with your result for part d)? [Hint: it should!]

▶ solution

a) $f(xyz) = 5xz - x^2y^3z^4$

$f_x = 5z - 2xy^3z^4 = \partial f/\partial x$

$f_y = -3x^2y^2z^4 = \partial f/\partial y$

$f_z = 5x - 4x^2y^3z^3 = \partial f/\partial z$

$f_x(-1,1,1) = 5(1) - 2(-1)1^3 1^4 = \boxed{7}$

$f_y(-1,1,1) = -3(-1)^2 1^2 1^4 = \boxed{-3}$

$f_z(-1,1,1) = 5(-1) - 4(-1)^2 1^3 1^3 = \boxed{-9}$

$f(-1,1,1) = 5(-1)(1) - (-1)^2 1^3 1^4 = \boxed{-6}$

b) $\boxed{L(x,y,z) = f(-1,1,1) + f_x(-1,1,1)(x+1) + f_y(-1,1,1)(y-1) + f_z(-1,1,1)(z-1) = -6 + 7(x+1) - 3(y-1) - 9(z-1)}$

$f(-0.99, 0.98, 1.01) \approx L(-0.99, 0.98, 1.01)$

$= -6 + 7(-0.99+1) - 3(0.98-1) - 9(1.01-1)$

$= -6 + 7(0.01) - 3(-0.02) - 9(0.01)$

$= -6 + 0.07 + 0.06 - 0.09$

$= -6 + 0.04 = \boxed{-5.96}$

[compare: $f(-.99, .98, 1.01) \approx -5.959$]

c) $f_{xy} = \dfrac{\partial}{\partial y}(5z - 2xy^3z^4)$

$= 0 - 6xy^2z^4$

$f_{xyz} = \dfrac{\partial}{\partial z}(-6xy^2z^4) = \boxed{-24xy^2z^3}$

$f_{xyz}(-1,1,1) = -24(-1)1^2 1^3 = \boxed{24}$

d) $\dfrac{\partial}{\partial x}\left[5xz - x^2y^3z^4 = -6\right]$

$5\underbrace{\dfrac{\partial}{\partial x}(xz)}_{1(z) + x\frac{\partial z}{\partial x}} - y^3 \underbrace{\dfrac{\partial}{\partial x}(x^2z^4)}_{2xz^4 + x^2\left(4z^3\frac{\partial z}{\partial x}\right)} = 0$

$5z + 5x\dfrac{\partial z}{\partial x} - 2xy^3z^4 - 4x^2y^3z^3\dfrac{\partial z}{\partial x} = 0$

$(5x - 4x^2y^3z^3)\dfrac{\partial z}{\partial x} = 2xy^3z^4 - 5z$

$\dfrac{\partial z}{\partial x} = \dfrac{2xy^3z^4 - 5z}{5x - 4x^2y^3z^3}$

$\left.\dfrac{\partial z}{\partial x}\right|_{(-1,1,1)} = \dfrac{2(-1) - 5}{5(-1) - 4(-1)^2} = \dfrac{-7}{-9} = \boxed{\dfrac{7}{9}}$

[e) $\dfrac{\partial}{\partial x}\left[-6 + 7(x+1) - 3(y-1) - 9(z-1) = -6\right]$

$7 - 9\dfrac{\partial z}{\partial x} = 0 \rightarrow \dfrac{\partial z}{\partial x} = \dfrac{7}{9}$ ✓]