MAT2500-03/04 09S Quiz 3   Print Name (Last, First) ______________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

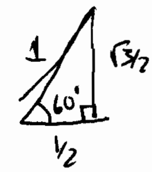
Given the vector-valued function $\vec{r}(t) = \langle \cos(t), \sin(t), \ln(\cos(t)) \rangle$ for the domain $-\frac{\pi}{2} < t < \frac{\pi}{2}$:

a) Evaluate $\vec{r}\,'(t)$, $\vec{r}\,''(t)$, $|\vec{r}\,'(t)|$, $\vec{T}(t)$ and remember to simplify your results (no credit for unidentified expressions)

b) Evaluate $\vec{r}\left(\frac{\pi}{3}\right)$, $\vec{r}\,'\left(\frac{\pi}{3}\right)$, $\vec{r}\,''\left(\frac{\pi}{3}\right)$, $\vec{T}\left(\frac{\pi}{3}\right)$ and remember to simplify your results (no credit for unidentified expressions).

c) Evaluate the exact angle θ in radians between $\vec{r}\,'\left(\frac{\pi}{3}\right)$ and $\vec{r}\,''\left(\frac{\pi}{3}\right)$ and a single decimal place approximation in degrees.

d) Evaluate the vector $\vec{w}$ which is the vector projection of $\vec{r}\,''\left(\frac{\pi}{3}\right)$ orthogonal to $\vec{r}\,'\left(\frac{\pi}{3}\right)$.



▶ solution

a) $\vec{r}(t) = \langle \cos(t), \sin(t), \ln(\cos(t)) \rangle$

$\vec{r}\,'(t) = \langle -\sin t, \cos t, -\frac{\sin t}{\cos t} \rangle$ $\tan t$

$\vec{r}\,''(t) = \langle -\cos t, -\sin t, -\sec^2 t \rangle$

$|\vec{r}\,'(t)| = \sqrt{\sin^2 t + \cos^2 t + \tan^2 t}$

$= \sqrt{1+\tan^2 t} = \sqrt{1+\frac{\sin^2 t}{\cos^2 t}} = \sqrt{\frac{\cos^2 t + \sin^2 t}{\cos^2 t}}$

$= |\sec t| = \sec t \quad (-\frac{\pi}{2} < t < \frac{\pi}{2})$

$\hat{T}(t) = \frac{\vec{r}\,'(t)}{|\vec{r}\,'(t)|} = \frac{\langle -\sin t, \cos t, -\tan t \rangle}{\sec t}$

$= \langle -\cos t \sin t, \cos^2 t, \sin t \rangle$

b) $\vec{r}(\frac{\pi}{3}) = \langle \cos\frac{\pi}{3}, \sin\frac{\pi}{3}, \ln\cos\frac{\pi}{3} \rangle = \langle \frac{1}{2}, \frac{\sqrt{3}}{2}, \ln(\frac{1}{2}) \rangle$

$\vec{r}\,'(\frac{\pi}{3}) = \langle -\frac{\sqrt{3}}{2}, \frac{1}{2}, -\sqrt{3} \rangle$

$\vec{r}\,''(\frac{\pi}{3}) = \langle -\frac{1}{2}, -\frac{\sqrt{3}}{2}, -4 \rangle$

$\hat{T}(\pi/3) = \langle -\frac{\sqrt{3}}{4}, \frac{1}{4}, -\frac{\sqrt{3}}{2} \rangle$

doing mental arithmetic while watching Grey's Anatomy is a sure recipe for human error

c) $|\vec{r}\,''(\frac{\pi}{3})| = \frac{1}{2}\sqrt{1+3+64} = \frac{1}{2}\sqrt{68} = \sqrt{17}$

$\widehat{\vec{r}\,''(\frac{\pi}{3})} = \frac{1}{2\sqrt{17}}\langle -1, -\sqrt{3}, -8 \rangle$

$\cos\theta = \hat{T}(\frac{\pi}{3}) \cdot \widehat{\vec{r}\,''(\frac{\pi}{3})} =$

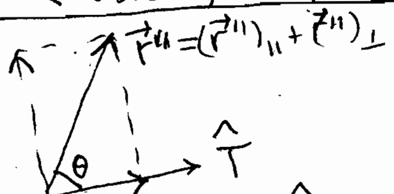
$= \frac{1}{4}\langle -\sqrt{3}, 1, -2\sqrt{3} \rangle \cdot \frac{\langle -1, -\sqrt{3}, -8 \rangle}{2\sqrt{17}}$

$= \frac{1}{8\sqrt{17}}(\sqrt{3} - \sqrt{3} + 16\sqrt{3}) = 2\sqrt{\frac{3}{17}} = 2\sqrt{\frac{3}{17}}$

$\theta = \arccos(2\sqrt{\frac{3}{17}}) \approx \boxed{32.8^\circ}$

oops my white-out pen ran dry! drat!

d) $\vec{w}$



$(\vec{r}\,'')_{\parallel} = (\vec{r}\,'' \cdot \hat{T})\hat{T}$

$\vec{r}\,''(\frac{\pi}{3})_{\parallel} = \underbrace{\hat{T}(\frac{\pi}{3}) \cdot \vec{r}\,''(\frac{\pi}{3})} \; \hat{T}(\pi/3) = 2\sqrt{3}\,\frac{1}{4}\langle -\sqrt{3}, 1, -2\sqrt{3} \rangle$

$\frac{1}{2}\frac{1}{4}(\sqrt{3} - \sqrt{3} + 16\sqrt{3}) \quad = \frac{1}{2}\langle -3, \sqrt{3}, -6 \rangle$

$= 2\sqrt{3}$

$\vec{w} = \vec{r}\,''(\frac{\pi}{3})_{\perp} = \vec{r}\,''(\frac{\pi}{3}) - \vec{r}\,''(\frac{\pi}{3})_{\parallel} = \frac{1}{2}\langle -1, -\sqrt{3}, -8 \rangle - \frac{1}{2}\langle -3, \sqrt{3}, -6 \rangle = \frac{1}{2}\langle 2, -2\sqrt{3}, -2 \rangle$

$= \boxed{\langle 1, -\sqrt{3}, -1 \rangle}$