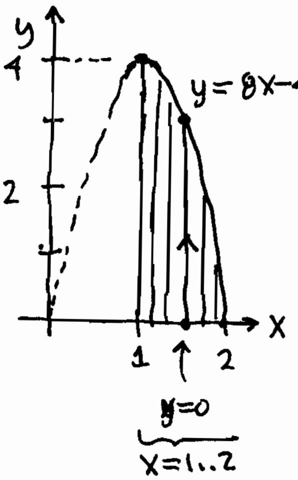
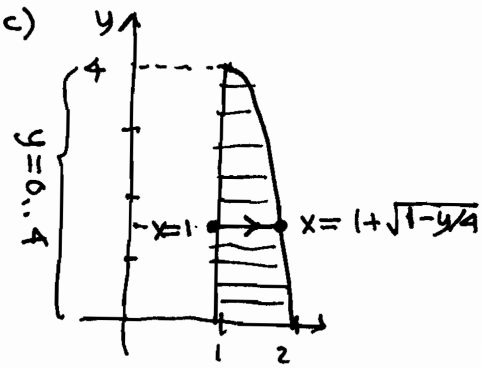
MAT 2500-03/04 09S Takehome Test 3 Answers (1)

① a) $\int_1^2 \underbrace{\int_0^{8x-4x^2} x\,dy}_{xy\,\big|_{y=0}^{y=8x-4x^2}}\,dx = \int_1^2 x(8x-4x^2)\,dx = \int_1^2 8x^2-4x^3\,dx = 8\frac{x^3}{3}-\cancel{4}\frac{x^4}{\cancel{4}}\Big|_1^2$

$= \frac{8}{3}(8-1)-(16-1) = \frac{56}{3}-\frac{15\cdot 3}{3} = \boxed{\frac{11}{3}}$

b) $\int_{x=1}^{x=2}\int_{y=0}^{y=8x-4x^2} x\,dy\,dx$

c) 

c) 

$y = 8x-4x^2 = 4x(x-2) = 0$
$\hookrightarrow x = 0, 2$

parabola vertex at $x=1 \to y=4$

c) solve for x:

$4x^2-8x+y=0$

$x = \frac{8\pm\sqrt{64-4(4)y}}{2\cdot 4} = 1\pm\sqrt{1-y/4}$

plus root between 1 & 2.

d) $\int_0^4\int_1^{1+\sqrt{1-y/4}} x\,dx\,dy \overset{e)}{=} \int_0^4 \frac{x^2}{2}\Big|_{x=1}^{x=1+\sqrt{1-y/4}} dy = \int_0^4 \frac{1}{2}\Big[\underbrace{\left(1+\sqrt{1-y/4}\right)^2}_{1+2\sqrt{1-y/4}+1-\frac{y}{4}}-1\Big]dy$

$= \int_0^4 \frac{1}{2}-\frac{y}{8}\,dy + \int_0^4 \underbrace{\left(1-\frac{y}{4}\right)^{1/2}}_{u=1-\frac{y}{4},\ du=-dy/4} dy = -4du$

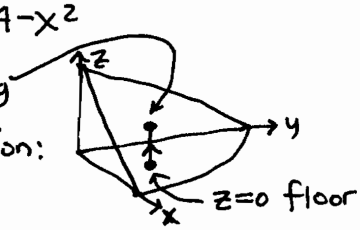
$\to \int u^{1/2}(-4du) = -4\frac{u^{3/2}}{3/2} = -\frac{8}{3}\left(1-\frac{y}{4}\right)^{3/2}$

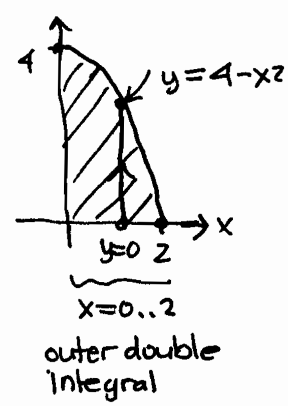
$= \frac{y}{2}-\frac{y^2}{16}-\frac{8}{3}\left(1-\frac{y}{4}\right)^{3/2}\Big|_0^4 = 2-1-0+\frac{8}{3} = \boxed{\frac{11}{3}}$ ✓ f) agrees! yes!

② a) $x^2+y+2z=4 \xrightarrow{z=0} x^2+y=4 \to y=4-x^2$

b) $\hookrightarrow$ solve for z: $z = \frac{4-x^2-y}{2} = 2-\frac{x^2}{2}-\frac{y}{2}$ ceiling

inner integration:



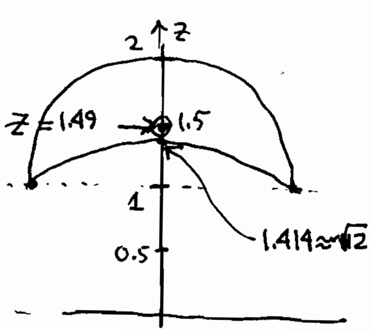


b) $V_z = \int_0^2\int_0^{4-x^2}\int_0^{2-\frac{x^2}{2}-\frac{y}{2}} z\,dz\,dy\,dx$

maple $\boxed{\frac{256}{105}} \approx 2.438095238$

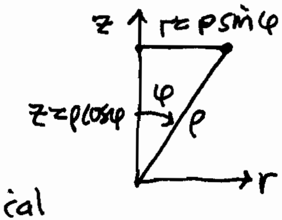
c) $256 = 2(105)+46$ ✓ hint works.

③ f) continued

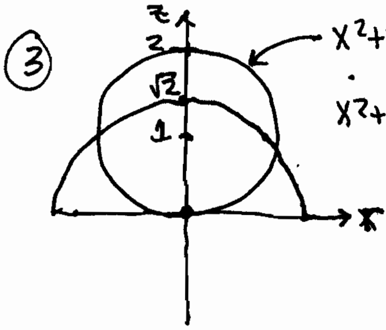


centroid is just inside solid region on z-axis

MAT2500-03/04 09S Takehome Test 3 Answers (2)



(3)



$x^2+y^2+(z-1)^2=1$ center (0,0,1) radius 1.

$x^2+y^2+z^2=2$ radius $\sqrt{2}\approx 1.414$.

a) $\underbrace{x^2+y^2}_{r^2}+(z-1)^2=1 \to$ cylindrical $\boxed{r^2+(z-1)^2=1}$ $\to \underbrace{r^2+z^2}_{\rho^2}-\underbrace{2z}_{\rho\cos\varphi}+\cancel{1}=\cancel{1}$ $\to \rho^2=\rho 2\cos\varphi$

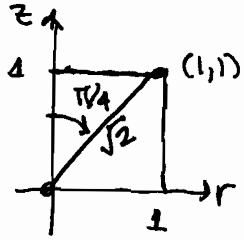
$\overbrace{x^2+y^2}^{r^2}+z^2=2 \to$ $\boxed{\underbrace{r^2+z^2}_{\rho^2}=2}$

spherical $\boxed{\begin{array}{l}\rho=2\cos\varphi\\ \rho=\sqrt{2}\end{array}}$

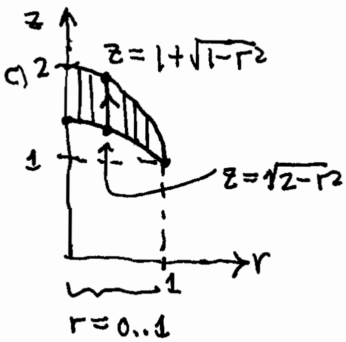
b) $r^2+z^2=2$
$-(r^2+z^2-2z+1=1)$
$2z-1=2-1$
$2z=2$
$z=1$

$\to r=\sqrt{2-z^2}=\sqrt{2-1}=1$

$\boxed{(r,z)=(1,1)}$



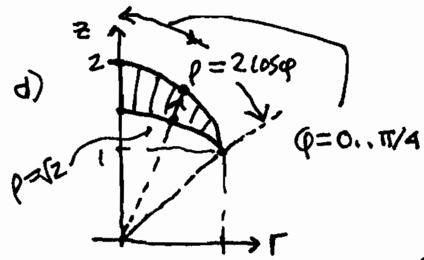
$\boxed{\begin{array}{l}\varphi=\pi/4\\ \rho=\sqrt{2}\end{array}}$ obvious from 45-90-45 right triangle.



solve for z:

$r^2+z^2=2 \to z=\pm\sqrt{2-r^2} \to \sqrt{2-r^2}$

$r^2+(z-1)^2=1 \to (z-1)^2=1-r^2 \to z-1=\pm\sqrt{1-r^2} \to z=1\overset{\text{root}>1}{+}\sqrt{1-r^2}$



c) d)
($\theta=0..2\pi$ to revolve around z-axis)

e) $V=\int_0^{2\pi}\int_0^1\int_{\sqrt{2-r^2}}^{1+\sqrt{1-r^2}} 1\,(r)\,dz\,dr\,d\theta \overset{f)}{=} \int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_{\sqrt{2}}^{2\cos\varphi} 1\,(\rho^2\sin\phi)\,d\rho\,d\phi\,d\theta \overset{\text{Maple}}{=} \frac{\pi}{3}(7-4\sqrt{2}) \approx 1.40654$

$Vz=\int_0^{2\pi}\int_0^1\int_{\sqrt{2-r^2}}^{1+\sqrt{1-r^2}} z\,(r)\,dz\,dr\,d\theta = \int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_{\sqrt{2}}^{2\cos\varphi} (\rho\cos\phi)(\rho^2\sin\phi)\,d\rho\,d\phi\,d\theta = \frac{2\pi}{3}\approx 2.09440$

$\bar{z}=Vz/V=\boxed{2/(7-4\sqrt{2})\approx 1.48904}>1.414$

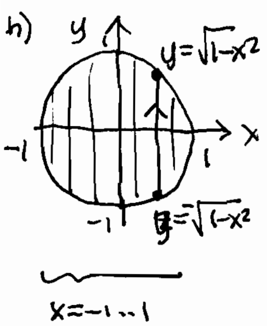
g) $V=\int_0^{2\pi}\int_0^{\frac{\pi}{4}} \frac{\rho^3}{3}\Big|_{\rho=\sqrt{2}}^{\rho=2\cos\varphi}\sin\phi\,d\phi\,d\theta=\underbrace{\int_0^{2\pi}1\,d\theta}_{2\pi}\int_0^{\frac{\pi}{4}}\left(\frac{8}{3}\underbrace{\cos^3\phi}_{u^3}-\frac{2\sqrt{2}}{3}\right)\underbrace{\sin\phi\,d\phi}_{-du}$

$\hookrightarrow -\frac{8}{3}\frac{u^4}{4}=-\frac{2}{3}\cos^4\phi$

$=2\pi\left[-\frac{2}{3}\cos^4\phi+\frac{2\sqrt{2}}{3}\cos\phi\right]_0^{\pi/4}$

$=2\pi\left[-\frac{2}{3}\left(\frac{1}{4}-1\right)+\frac{2\sqrt{2}}{3}\left(\frac{1}{\sqrt{2}}-1\right)\right]=2\pi\left[\frac{7}{6}-\frac{2\sqrt{2}}{3}\right]=\boxed{\pi\left(\frac{7}{3}-\frac{4\sqrt{2}}{3}\right)}\approx 1.406538942$

h)



$\begin{pmatrix}V\\ Vz\end{pmatrix}=\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{\sqrt{2-x^2-y^2}}^{1+\sqrt{1-x^2-y^2}}\begin{pmatrix}1\\ z\end{pmatrix}dz\,dy\,dx$ $\overset{\text{Maple}}{=}$ same results as above (simplified)

z-limits from above: $z=\sqrt{2-r^2}\,..\,1+\sqrt{1-r^2} \to z=\sqrt{2-x^2-y^2}\,..\,1+\sqrt{1-x^2-y^2}$