Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC, MathCad).
For each hand integration step, state the antiderivative formula used before substituting limits into it:

$\int_a^b f(x)\ dx = F(x)|_{x=a}^{x=b} = F(b) - F(a)$. Explain in as many words as possible everything you are doing! Make explanatory diagrams wherever possible.

1. $\int_1^2 \int_0^{8x-4x^2} x\ dy\ dx$ a) Evaluate this exactly by hand step by step (easy!) and by technology.

b) Make a completely labeled diagram of the region of integration with a typical labeled cross-section representing the current iteration of the integral.
c) Make a new completely labeled diagram corresponding to the reversed order of integration.
d) State the new integral with the order of integration reversed.
e) Evaluate the new integral by exactly by hand (you need a simple u-substitution) and using technology.
f) Do you get the same result as in part a)?

2. Consider the solid region R in the first octant between the coordinate planes (sides and bottom) $x=0$, $y=0$, $z=0$ and the surface (top) $x^2 + y + 2\ z = 4$.
a) Where does this surface intersect the x-y plane? Make a labeled diagram of the floor of this solid region in that plane.
b) Set up and evaluate exactly using technology the integral $Vz = \iiint z\ dV$ over this solid region, explaining how you got your limits of integration, and add a typical endpoint labeled cross-section to your diagram of part a) that explains the outer double integral.

c) The exact answer is a rational number $\dfrac{n}{d}$ which satisfies $n = 2\ d + 46$. Does your result agree with this?
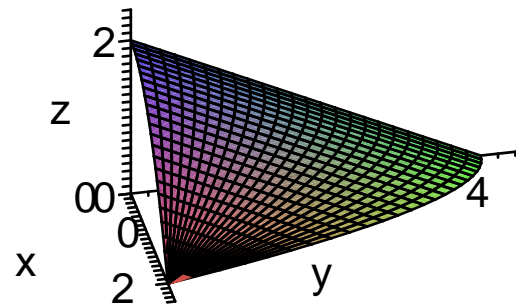

3. Consider the solid of revolution R between the two spheres $x^2 + y^2 + (z-1)^2 = 1$ and $x^2 + y^2 + z^2 = 2$ whose cross-section is given in the figure (next page).
a) Express the equation for these two surfaces first in cylindrical coordinates and then in spherical coordinates. (Simplify.)

b) Find the cylindrical coordinates (r, z) of the circle of intersection of the two spheres (eliminate r^2 from their equations, solve for z, then find r), and then the spherical coordinates (ρ, ϕ) of this circle.

c) Make a new r-z diagram illustrating a typical cylindrical coordinate vertical cross-section, labeling its endpoints by the starting and stopping values of z and describe the range of values of the remaining coordinates over this region.

d) Repeat for spherical coordinates, a typical radial cross-section and its radial coordinate ρ.
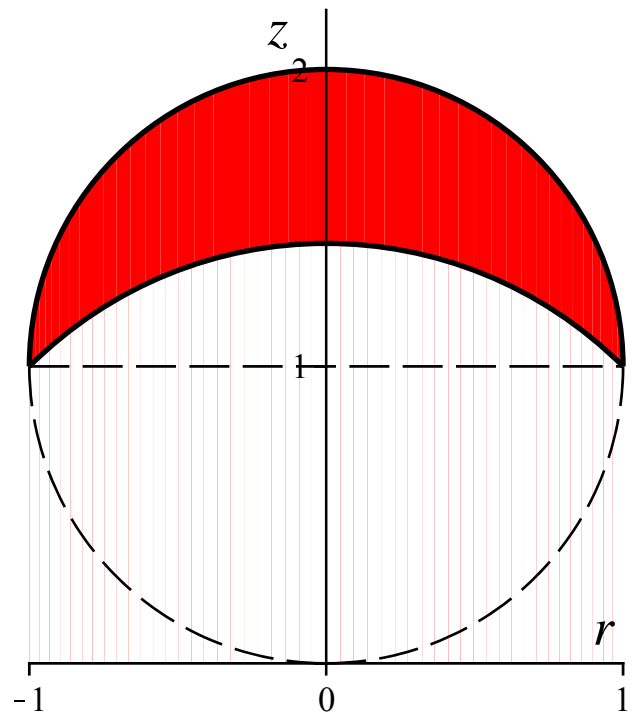
e) Express the triple integrals $V = \iiint_R 1\ dV,\ \iiint_R z\ dV$ in cylindrical coordinates, and evaluate the integral exactly with technology, then to 5 decimal place accuracy. Evaluate their ratio $Z = V^{-1} \iiint_R z\ dV$ for the location of the centroid of the solid along the z-axis to 4 decimal place accuracy.

f) Repeat for spherical coordinates. Your results should agree with part e). Locate the position on the axis of the centroid on the test sheet diagram. Does it lie inside or outside the solid region?

g) Evaluate the volume exactly step by step by hand in spherical coordinates, where the antiderivatives are all straightforward (using a simple u-substitution for the φ integration).

h) As a final check, express the above two integrals in Cartesian coordinates (supporting your work with fully labeled cross-section diagrams) and evaluate them with Maple to confirm your previous values. [Be patient when Maple issues a warning about the volume integral; it eventually gives the correct result!]

▶ solution (on-line)

▼ pledge