MAT2500-03/04 09S TEST 2 Answers

① a) $f(x,y) = x^3 - 6xy + y^2$

$f_x(x,y) = 3x^2 - 6y$

$f_y(x,y) = -6x + 2y$

a) $f_x(6,18) = 3\cdot 6^2 - 6\cdot 18 = 0$ ✓

$f_y(6,18) = -6(6) + 2(18) = 0$ ✓

$\therefore$ (6,18) is a critical pt of f

b)

	(0,0)	(6,18)
$f_{xx}(x,y) = 6x$	0 ?	$36>0$ $\smile$ local min
$f_{yy}(x,y) = 2$	$2>0$ $\smile$ local min	$2>0$ $\smile$ local min
$f_{xy}(x,y) = -6$	-6	-6
$f_{xx}f_{yy} - f_{xy}^2$	$0(2)-(-6)^2<0$ $\boxed{\text{saddle}}$	$36(2)-(-6)^2>0$ confirms $\boxed{\text{local min}}$

② a) $z = 3x^2 - y^2 + 2x = f(x,y)$ $\begin{cases} f_x = 6x+2 \\ f_y = -2y \end{cases}$

linear approx: $L(x,y) = \underbrace{f(1,-2)}_{1} + \underbrace{f_x(1,-2)}_{8}(x-1) + \underbrace{f_y(1,-2)}_{4}(y+2)$

$f(1,-2) = 3(1)^2 - (-2)^2 + 2(1) = 1$

$f_x(1,-2) = 6(1)+2 = 8$

$f_y(1,-2) = -2(-2) = 4$

tangent plane is graph of linear approx:

$\boxed{z = 1 + 8(x-1) + 4(y+2)}$

or $z = 1 + 8x + 4y - 8 + 8$

$\boxed{8x + 4y - z = -1}$ $\longrightarrow \vec{n} = \langle 8,4,1\rangle$

OR: $z - 3x^2 + y^2 - 2x = 0$

$F(x,y,z)$

$\vec{\nabla}F(x,y,z) = \langle -6x-2, 2y, 1\rangle$

$\vec{\nabla}F(1,-2,1) = \langle -8,-4,1\rangle = \vec{n}$

$\vec{r}_0 = \langle 1,-2,1\rangle$

$\vec{n}\cdot(\vec{r}-\vec{r}_0) = 0$

$\langle -8,-4,1\rangle \cdot \langle x-1, y+2, z-1\rangle = 0$

$-8(x-1) - 4(y+2) + (z-1) = 0$

$\boxed{-8x - 4y + z = 1}$

② b) $f(1,-2) = 1$ (given)

level curve: $\boxed{3x^2 - y^2 + 2x = 1}$

c) $L(x,y) = 1 + 8(x-1) + 4(y+2)$ (above)

$f(1.01,-1.98)$ $(= 1.1599)$

$\approx L(1.01,-1.98) = 1 + 8(\underbrace{1.01-1}_{.01}) + 4(\underbrace{-1.98+2}_{.02})$

$= 1 + .08 + .08 = \boxed{1.16}$

③ a) $F(x,y,z) = x^2 + 2y^2 - 3z^2 = 3$

$\vec{\nabla}F(x,y,z) = \langle 2x, 4y, -6z\rangle$

$\vec{\nabla}F(2,-1,1) = \langle 4,-4,-6\rangle = 2\underbrace{\langle 2,-2,-3\rangle}_{\vec{n}}$

$\vec{r}_0 = \langle 2,-1,1\rangle$

$\vec{r} = \vec{r}_0 + t\vec{n} = \langle 2,-1,1\rangle + t\langle 2,-2,-3\rangle$

$\boxed{\langle x,y,z\rangle = \langle 2+2t, -1-2t, 1-3t\rangle}$

(OR $= \langle 2+4t, -1-4t, 1-6t\rangle$)

b) $\vec{PO} = \langle 0,0,0\rangle - \langle 2,-1,1\rangle = \langle -2,1,-1\rangle$

$\hat{u} = \widehat{PQ} = \dfrac{\langle -2,1,-1\rangle}{\sqrt{6}}$

$\vec{\nabla}F(2,-1,1) = \langle 2,-4,-6\rangle$

$D_{\hat{u}}F(2,-1,1) = \hat{u}\cdot\vec{\nabla}F(2,-4,-6)$

$= \dfrac{1}{\sqrt{6}}\langle -2,1,-1\rangle\cdot\langle 4,-4,-6\rangle = \dfrac{-8-4+6}{\sqrt{6}}$

$= \boxed{-\dfrac{6}{\sqrt{6}} = -\sqrt{6}}$

c) $\widehat{\vec{\nabla}F}(2,-1,1) = \boxed{\dfrac{\langle 2,-2,-3\rangle}{\sqrt{17}}}$ direction of most rapid increase

$|\vec{\nabla}F(2,-1,1)| = 2\sqrt{4+4+9} = \boxed{2\sqrt{17}}$ maximum rate of change

d) $\dfrac{d}{dt}F(x(t),y(t),z(t)) = \dfrac{\partial F}{\partial x}(\ldots)\dfrac{dx}{dt} + \dfrac{\partial F}{\partial y}(\ldots)\dfrac{dy}{dt} + \dfrac{\partial F}{\partial z}(\ldots)\dfrac{dy}{dt}$

$t=1$: $x=1, y=1, z=1$, $\langle x',y',z'\rangle|_{t=1} = \langle 1, 2t, 3t^2\rangle|_{t=1} = \langle 1,2,3\rangle$

$\vec{\nabla}F(1,1,1) = \langle 2,4,-6\rangle$

$\left.\dfrac{dF}{dt}\right|_{t=1} = 2(1) + 4(2) - 6(3) = 2 + 8 - 18 = \boxed{-8}$

I admit even with simple numbers as I did the answer key, I made several stupid errors — which I caught with my Maple worksheet. When it really counts, these kinds of mechanics are better done with technology.