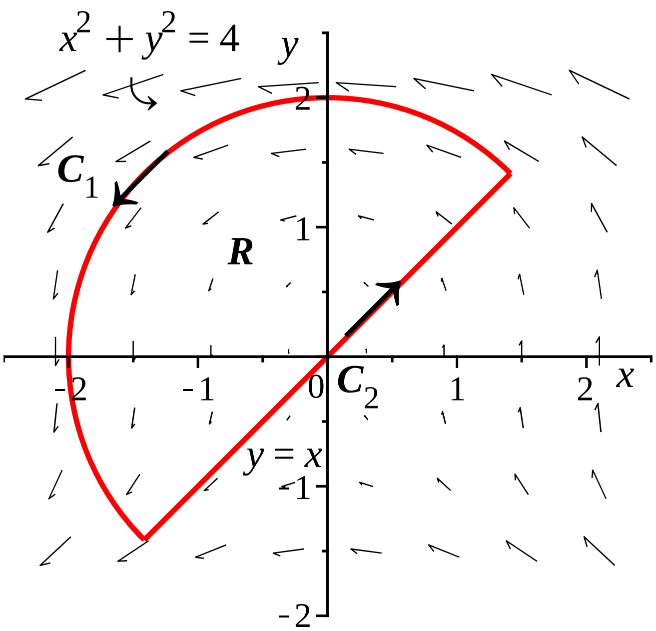
MAT2500-02 08S Final Exam   Print Name (Last, First) ________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).
You may use technology to evaluate any integrals you set up, as long as you can give the exact symbolic result.

1. Given the point $(x, y, z) = (3, 4, -12)$, find the new coordinates, in each case stating the angles both in radians (exactly, using inverse trig functions) and in degrees (1 decimal place accuracy) and use proper identifying symbols for all coordinates: a) cylindrical coordinates. b) spherical coordinates. Support your work with two diagrams, one of the xy plane and one of the rz half plane, each including a reference triangle locating the point with respect to the axes with all three sides labeled by their lengths and both axes labeled by their coordinate labels. Show clearly how you obtain values of your coordinates from these diagrams.



2. a) Describe the region $\boldsymbol{R}$ bounded by the closed counterclockwise directed curve $\boldsymbol{C} = \boldsymbol{C}_1 \cup \boldsymbol{C}_2$ by giving the appropriate intervals of the polar coordinates over the region, and draw in the diagram a typical radial cross-section, labeling its endpoints by the values of the radial coordinate.

b) Use polar coordinates to evaluate $A_y = \iint_{\boldsymbol{R}} y\, dA$ and $A = \iint_{\boldsymbol{R}} 1\, dA$. What is the average value of y over the region $\boldsymbol{R}$? The ratio $\bar{y} = A_y / A$ is the y coordinate of the centroid of the region. Does it seem right? Explain.

c) The vector field $\vec{\boldsymbol{F}} = \left\langle -\frac{y^2}{2}, x \right\rangle$ is shown in the diagram. Explain why its line integral around $\boldsymbol{C}$ should be positive or negative.

d) Evaluate the line integral of this vector field directly: $\oint_{\boldsymbol{C}} \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{r}}$. Give the exact value and its decimal approximation to at least several decimal places. Does it have the sign you said it should have in part c)?

e) Check your result by evaluating its equivalent value by Green's theorem: $\iint_{\boldsymbol{R}} \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\, dA$. Can you see how to use the results of part b) as an additional check on this integral? Explain and do so if you can.

3. a) Evaluate the curl and divergence of the vector fields $\vec{\boldsymbol{F}} = \langle 2xy, x^2 + 2yz, y^2 \rangle$ and $\vec{\boldsymbol{G}} = \langle y e^{-x}, e^{-x}, 2z \rangle$.
b) Which of these is a conservative vector field and why?

4. a) Evaluate the gradient vector field $\vec{\boldsymbol{F}} = \vec{\nabla} f$ associated with the potential function $f = 3x^2 + 2xy + 3y^2$.
b) Use the potential to evaluate the line integral $\int_C \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{r}}$ over any curve from $\left(-\frac{1}{2}, \frac{1}{2}\right)$ to $\left(\frac{1}{2}, \frac{1}{2}\right)$.

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Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).
For each hand integration step, state the antiderivative formula used before substituting limits into it:

$$\int_a^b f(x)\,dx = F(x)\big|_{x=a}^{x=b} = F(b) - F(a).$$

1. Given the point $(x, y, z) = (1, \sqrt{3}, 3)$, find the new coordinates, in each case stating the angles both in radians (exactly) and degrees and use proper identifying symbols for all coordinates: a) cylindrical coordinates. b) spherical coordinates.
Support your work with clearly labeled diagrams.

2. $\displaystyle\int_0^2 \int_0^{\sqrt{2x-x^2}} y\sqrt{x^2+y^2}\,dy\,dx.$

a) Make a fully labeled diagram illustrating the integration scheme including a cross-section indicating the inner integration and label its endpoints properly.
b) Convert this integral to polar coordinates, supplying a new diagram with the fully labeled radial cross-section.
c) Evaluate this last integral step by step by hand.
d) Optional check: does your result agree with both integrals evaluated using technology?

3. $\boldsymbol{F} = \langle y\cos(xy), x\cos(xy), -\sin(z)\rangle$
a) Evaluate the divergence of $\boldsymbol{F}$.
b) Show that $\boldsymbol{F}$ is a conservative vector field.
c) Find a potential function f for $\boldsymbol{F}$.
d) Use the potential function to evaluate the line integral $\int_C \boldsymbol{F}\cdot d\boldsymbol{r}$, where C is any curve from $(0, 0, 0)$ to $(1/2, \pi, \pi/3)$.
e) Check your result by evaluating this line integral along the straight line segment from $(0, 0, 0)$ to $(1/2, \pi, \pi/3)$. Compare your results.

4. a) Verify Green's Theorem $\oint_C \boldsymbol{F}\cdot d\boldsymbol{r} = \iint_D \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\,dA$ for the region D inside the circle $x^2+y^2=4$ above the x-axis and the vector field $\boldsymbol{F} = \langle x, xy\rangle$. [Namely, evaluate both sides of Green's Theorem!] Start by making a diagram of D and the directed closed curve C (semicircle plus diameter), indicating its direction. Evaluate the double integral in both Cartesian and polar coordinates.
b) Evaluate the coordinates $\langle \bar{x}, \bar{y}\rangle = L^{-1} < \int_C x\,ds, \int_C y\,ds >$ of the centroid of the closed curve C, where $L = \int_C 1\,ds$ is its length. Make a diagram of the curve and locate its centroid. Does it seem reasonable?