MAT2705-02/06 09F Quiz 5 Print Name (Last, First) ________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

1. $2x_1 + 2x_2 + x_3 = 0$

$x_1 + 3x_2 + x_4 = 0$ a) Write down the coefficient matrix **A**, the RHS matrix **b** and the augmented matrix **C** = <**A** | **b**> for this linear system of equations.

b) By hand in a few easy steps or with technology (identify your choice!), reduce this matrix **C** to its ReducedRowEchelonForm.

c) Write out the pair of equations that correspond to the reduced matrix. Identify the leading variables and the free variables and solve. State your solution in the scalar form: $x_1 = \ldots, x_2 = \ldots$, etc .

d) Now state your solution in column matrix ("vector") form $\boldsymbol{x} = \ldots$ and then re-express it as an arbitrary linear combination of fixed vectors by factoring out the vector of coefficients of each of the free parameters in the solution.

2. $2x_1 + 2x_2 = 4$

$x_1 + 3x_2 = 1$ a) Write this linear system in the matrix form $\mathbf{A}\boldsymbol{x} = \mathbf{b}$.

b) From the matrix C and its row reduced form of the previous problem (ignoring the last zero column, so that we have augmented the matrix of the first two columns by the identity matrix), identify the inverse matrix for the matrix **A** of coefficients of this linear system.

c) Now solve this linear system using matrix multiplication by the inverse matrix.

d) Check by backsubstitution into the original two equations that your solution is a solution.

▶ solution

① a)

$$\underbrace{\begin{bmatrix} 2 & 2 & 1 & 0 \\ 1 & 3 & 0 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}}_{\vec{x}} = \underbrace{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}_{\vec{b}}$$

$$C = \begin{bmatrix} 2 & 2 & 1 & 0 & 0 \\ 1 & 3 & 0 & 1 & 0 \end{bmatrix} \xrightarrow{b)} R_1 \leftrightarrow R_2$$

$$\begin{bmatrix} 1 & 3 & 0 & 1 & 0 \\ 2 & 2 & 1 & 0 & 0 \\ -2 & -6 & -0 & -2 & -0 \end{bmatrix} \rightarrow R_2 \rightarrow R_2 - 2R_1$$

$$\begin{bmatrix} 1 & 3 & 0 & 1 & 0 \\ 0 & -4 & 1 & -2 & 0 \end{bmatrix} \rightarrow R_2 \rightarrow -\tfrac{1}{4}R_2$$

$$\begin{bmatrix} -0 & -3 & +3/4 & -3/2 & -0 \\ 1 & 3 & 0 & 1 & 0 \\ 0 & 1 & -\frac{1}{4} & \frac{1}{2} & 0 \end{bmatrix} \rightarrow R_1 \rightarrow R_1 - 3R_2$$

$$\begin{array}{c} \begin{bmatrix} 1 & 0 & 3/4 & -1/2 & 0 \\ 0 & 1 & -1/4 & \frac{1}{2} & 0 \end{bmatrix} \\ \begin{matrix} x_1 & x_2 & x_3 & x_4 \\ L & L & F & F \end{matrix} \end{array} \rightarrow \boxed{\begin{aligned} x_1 &= -\tfrac{3}{4}t_1 + \tfrac{1}{2}t_2 \\ x_2 &= \tfrac{1}{4}t_1 - \tfrac{1}{2}t_2 \\ x_3 &= t_1 \\ x_4 &= t_2 \end{aligned}}$$

c) $$\boxed{\begin{aligned} x_1 + \tfrac{3}{4}x_3 - \tfrac{1}{2}x_4 &= 0 \\ x_2 - \tfrac{1}{4}x_3 + \tfrac{1}{2}x_4 &= 0 \end{aligned}}$$

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -3/4\,t_1 + 1/2\,t_2 \\ 1/4\,t_1 - 1/2\,t_2 \\ t_1 \\ t_2 \end{bmatrix} = t_1 \begin{bmatrix} -3/4 \\ 1/4 \\ 1 \\ 0 \end{bmatrix} + t_2 \begin{bmatrix} 1/2 \\ -1/2 \\ 0 \\ 1 \end{bmatrix}$$

② a)

$$\overbrace{\begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}}^{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 2 & 1 & 0 \\ 1 & 3 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 3/4 & -1/2 \\ 0 & 1 & -1/4 & 1/2 \end{bmatrix}$$

$$\underbrace{\begin{matrix} 3/4 & -1/2 \\ -1/4 & 1/2 \end{matrix}}_{A^{-1}}$$

b) $$A^{-1}[A\vec{x} = \vec{b}] \rightarrow \vec{x} = A^{-1}\vec{b} = \frac{1}{4}\begin{bmatrix} 3 & -2 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 12-2 \\ -4+2 \end{bmatrix} = \boxed{\begin{bmatrix} 5/2 \\ -1/2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}$$

d) $2(\tfrac{5}{2}) + 2(-\tfrac{1}{2}) = 4$?

$= 5 - 1$ ✓

$= 4$

$(\tfrac{5}{2}) + 3(-\tfrac{1}{2}) = 1$?

$= \tfrac{5}{2} - \tfrac{3}{2}$

$= 1$ ✓