MAT2705-02/06 09F Quiz 4 Print Name (Last, First) ______________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

$2\,x_1 + 2\,x_3 - 2\,x_4 = 4$
$3\,x_1 + x_2 + x_3 - 3\,x_4 = 7$
$2\,x_1 - x_2 + 4\,x_3 - 2\,x_4 = 3$

1a) Write down the coefficient matrix A, the RHS matrix $\mathbf{b}$ and the augmented matrix $C = \langle A|\mathbf{b}\rangle$ for this linear system of equations.
b) By hand reduce this matrix to its *ReducedRowEchelonForm* in a few easy steps, annotating the operations you apply to each successive matrix in the process, using the notation: $R_1 \to 3\,R_1, R_1 \to R_1 + 2\,R_2, R_1 \leftrightarrow R_2$.
[You may use technology to check your steps.]
c) Write out the equations that correspond to the reduced matrix. Identify the leading variables ("L") and the free variables ("F") and solve the system according to the accepted procedure. State your solution in the form: $x_1 = \ldots, x_2 = \ldots, x_3 = \ldots, x_4 = \ldots$
d) Check by backsubstitution of your solution into the first equation of the original system that it is satisfied.

2.

$$\begin{bmatrix} 1 & 2 & 0 & 0 & 0 & 3 \\ 0 & 0 & 1 & 1 & 0 & 4 \\ 0 & 0 & 0 & 0 & 1 & -5 \end{bmatrix}$$

This matrix is the reduced matrix for a 3x5 linear system in the variables x_1, x_2, x_3, x_4, x_5. Identify the leading variables ("L") and the free variables ("F"). Write out the equations that correspond to the reduced matrix and solve the system, stating your solution as above.

▶ solution

① a)
$2x_1 + 0x_2 + 2x_3 - 2x_4 = 4$
$3x_1 + 1x_2 + 1x_3 - 3x_4 = 7$
$2x_1 - 1x_2 + 4x_3 - 2x_4 = 3$

$$A = \begin{bmatrix} 2 & 0 & 2 & -2 \\ 3 & 1 & 1 & -3 \\ 2 & -1 & 4 & -2 \end{bmatrix} \qquad \vec{b} = \begin{bmatrix} 4 \\ 7 \\ 3 \end{bmatrix}$$

$$C = \begin{bmatrix} 2 & 0 & 2 & -2 & 4 \\ 3 & 1 & 1 & -3 & 7 \\ 2 & -1 & 4 & -2 & 3 \end{bmatrix} \xrightarrow{\text{b)}} R_1 \to \tfrac{1}{2} R_1$$

$$\begin{bmatrix} \boxed{1} & 0 & 1 & -1 & 2 \\ 3 & 1 & 1 & -3 & 7 \\ 2 & -1 & 4 & -2 & 3 \end{bmatrix} \longrightarrow \begin{matrix} R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 - 2R_1 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & -1 & 2 \\ 0 & \boxed{1} & -2 & 0 & 1 \\ 0 & -1 & 2 & 0 & -1 \end{bmatrix} \longrightarrow R_3 \to R_3 + R_2$$

$$\begin{array}{c} \begin{bmatrix} 1 & 0 & 1 & -1 & 2 \\ 0 & 1 & -2 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\ \begin{matrix} x_1 & x_2 & x_3 & x_4 \\ L & L & F & F \end{matrix} \end{array}$$

c)
$$\boxed{\begin{aligned} x_1 + x_3 - x_4 &= 2 \\ x_2 - 2x_3 &= 1 \\ 0 &= 0 \end{aligned}}$$

$x_1 = 2 - x_3 + x_4 = 2 - t_1 + t_2$
$x_2 = 1 + 2x_3 = 1 + 2t_1$

$x_3 = t_1, x_4 = t_2$

$\therefore \boxed{x_1 = 2 - t_1 + t_2,\ x_2 = 1 + 2t_1,\ x_3 = t_1,\ x_4 = t_2}$

d) $2(2 - t_1 + t_2) + 2(t_1) - 2(t_2) = 4\ ?$
$4 \cancel{- 2t_1} \cancel{+ 2t_2} \cancel{+ 2t_1} \cancel{- 2t_2} \qquad 4 = 4$ ✓

②
$$\begin{array}{c} \begin{matrix} L & F & L & F & L \\ x_1 & x_2 & x_3 & x_4 & x_5 \end{matrix} \\ \begin{bmatrix} 1 & 2 & 0 & 0 & 0 & 3 \\ 0 & 0 & 1 & 1 & 0 & 4 \\ 0 & 0 & 0 & 0 & 1 & -5 \end{bmatrix} \end{array}$$

leading: x_1, x_3, x_5
free: x_2, x_4

$x_1 + 2x_2 = 3 \to x_1 = 3 - 2t_1$
$x_3 + x_4 = 4 \to x_3 = 4 - t_2$
$x_5 = -5 \quad x_5 = -5$

$x_2 = t_1,\ x_4 = t_2$

$\therefore \boxed{x_1 = 3 - 2t_1,\ x_2 = t_1,\ x_3 = 4 - t_2,\ x_4 = t_2,\ x_5 = -5}$