MAT2705-02/06 09F Quiz 2 Print Name (Last, First) ______________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

1. $\frac{dy}{dx} = -2\,x\,y^2$

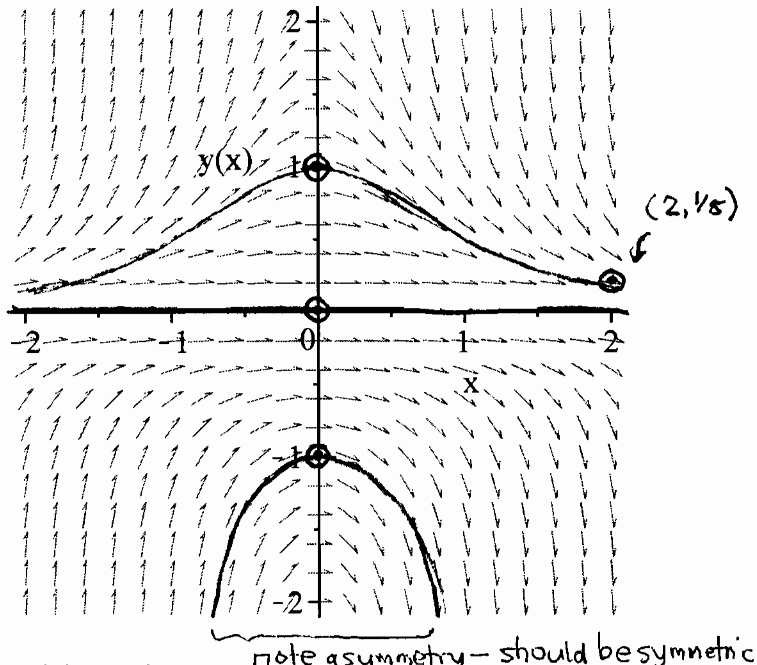
initial condition 1: $y(0) = 1$;
initial condition 2: $y(0) = 0$;
initial condition 3: $y(0) = -1$.

a) Indicate these initial data points on the graph by circled dots and roughly draw in the corresponding solution curves.

b) Find the (almost) general solution of the differential equation. What obvious solution is missing from this family?

c) Find the solution which satisfies the first initial condition.

d) Evaluate $y(2)$ for this solution and mark the corresponding point on the graph by a circled dot. Is this consistent with your approximate hand drawn solution? Explain.



e) Check by hand that your solution to c) solves the differential equation.

f) Enter the differential equation and its initial condition separated by a comma in Maple. Right click and solve. Write down the form of the solution that it gives you. Now right click and choose Simplify, simplify. Does it agree with your hand solution? Can you simplify Maple's solution by hand to get the same result? If so, show the steps.

▶ solution

① a) see graph. connecting up the arrows is not so easy. My lower curve is slightly off just based on the symmetry of the slope field — the soln curve should also be symmetric.

b) $\frac{dy}{dx} = -2xy^2$ divide by y^2 so $y \neq 0$

$\int y^{-2}dy = \int -2x\,dx$

$\frac{y^{-1}}{-1} = -2\left(\frac{x^2}{2}\right) + C$

$-\frac{1}{y} = -x^2 + C$

$\boxed{y = \frac{-1}{C-x^2} = \frac{1}{x^2-C}}$ general soln

but omits obvious constant soln $\boxed{y=0.}$

c) $1 = y(0) = \frac{1}{-C} \to C = -1$

so $\boxed{y = \frac{1}{x^2-(-1)} = \frac{1}{x^2+1}}$

d) $y(2) = \frac{1}{2^2+1} = \frac{1}{5}$ so point $(2, 1/5)$ lies on this solution curve.

Counting arrows, grid separation is 1/5 so this point is at center of arrow above (2,0). My hand drawn curve is slightly below but not bad considering the limitations of hand drawing.

e) $y = (x^2+1)^{-1}$

$\frac{dy}{dx} = -(x^2+1)^{-2}\cdot 2x = \frac{-2x}{(x^2+1)^2} \to \frac{dy}{dx} = -2xy^2$

$\frac{-2x}{(x^2+1)^2} = -2x\left(\frac{1}{x^2+1}\right)^2 = \frac{-2x}{(x^2+1)^2}$ ✓

hand simplification

f) Maple: $y = -\frac{1}{2}\,\frac{-2x^2+2}{-1+x^4} = \frac{x^2-1}{x^4-1} = \frac{\cancel{x^2-1}}{\cancel{(x^2-1)}(x^2+1)}$

$= \frac{1}{x^2+1}$ ✓ using $a^2-b^2 = (a-b)(a+b)$