

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

1. $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$, soln: $y(x) = e^{2x}$

a) Verify that $y(x)$ satisfies the given differential equation.

b) Determine all values of the constant k so that $y(x) = e^{kx}$ satisfies the differential equation; state the corresponding solutions $y(x)$.

Organize your work as though you were playing professor.

2. Write a differential equation that models the situation: "The time rate of change of the number N of big screen TV's sold per month is proportional to the product of that number and its difference with the saturation number N_{\max} ."

► solution

1. a) $y = e^{2x}$
 $y' = 2e^{2x}$
 $y'' = 4e^{2x}$

$$y'' - 3y' + 2y = 0$$

$$4e^{2x} - 3(2e^{2x}) + 2e^{2x} = 0$$

$$\underbrace{4e^{2x} - 6e^{2x} + 2e^{2x}}_{(4-6+2)e^{2x}} = 0$$

$$\underbrace{0}_{0} = 0 \checkmark$$

b) $y = e^{kx}$
 $y' = ke^{kx}$
 $y'' = k^2e^{kx}$

$$y'' - 3y' + 2y = 0$$

$$k^2e^{kx} - 3ke^{kx} + 2e^{kx} = 0$$

$$e^{kx}(k^2 - 3k + 2) = 0$$

$\neq 0$

$$\hookrightarrow k^2 - 3k + 2 = 0 \xrightarrow{\text{or Quad F}} k = \frac{3 \pm \sqrt{9 - 4 \cdot 2}}{2}$$

$$(k-1)(k-2) = 0 = \frac{3 \pm 1}{2} = \frac{4}{2}, \frac{2}{2}$$

$$= 2, 1$$

$k = 1, 2$
 $y = e^{kx} = e^x, e^{2x}$

2. $\frac{dN}{dt} \propto N(N - N_{\max}) \rightarrow$

$$\frac{dN}{dt} = kN(N - N_{\max})$$

(or $\frac{dN}{dt} = kN(N_{\max} - N)$)
 opposite sign for k