MAT2705-02/06 09F TEST2 Answers

① a) see test sheet graph (lower right)

graphically one finds $\vec{v}_3 = 2\vec{v}_1 + 4\vec{v}_2$

so $(y_1, y_2) = (2,4)$.

These coefficients are the number of times tip to tail one repeats each of the respective vectors to get the 2 edges of the parallelogram coming from the origin.

b) $\begin{bmatrix}2\\8\end{bmatrix} = y_1\begin{bmatrix}3\\2\end{bmatrix} + y_2\begin{bmatrix}-1\\1\end{bmatrix} = \begin{bmatrix}3 & -1\\2 & 1\end{bmatrix}\begin{bmatrix}y_1\\y_2\end{bmatrix}$

$\hookrightarrow \begin{bmatrix}y_1\\y_2\end{bmatrix} = \underbrace{\begin{bmatrix}3 & -1\\2 & 1\end{bmatrix}^{-1}}_{\frac{1}{5}\begin{bmatrix}1 & 1\\-2 & 3\end{bmatrix}}\begin{bmatrix}2\\8\end{bmatrix} = \frac{1}{5}\begin{bmatrix}2+8\\-4+24\end{bmatrix} = \frac{1}{5}\begin{bmatrix}10\\20\end{bmatrix} = \begin{bmatrix}2\\4\end{bmatrix}.$

$\boxed{\text{so } \begin{bmatrix}2\\8\end{bmatrix} = 2\begin{bmatrix}3\\2\end{bmatrix} + 4\begin{bmatrix}-1\\1\end{bmatrix}}$ c) $= \begin{bmatrix}6-4\\4+4\end{bmatrix} = \begin{bmatrix}2\\8\end{bmatrix}$ ✓ yes!

d) yes!

② a) $A = \begin{bmatrix}2 & 1 & -3 & 3\\1 & 2 & 0 & -2\\-1 & -3 & -1 & 2\end{bmatrix}$

$A\vec{x} = \vec{0}$: $\begin{bmatrix}2 & 1 & -3 & 3\\1 & 2 & 0 & -2\\-1 & -3 & -1 & 2\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$

$\langle A|\vec{0}\rangle = \begin{bmatrix}2 & 1 & -3 & 3 & 0\\1 & 2 & 0 & -2 & 0\\-1 & -3 & -1 & 2 & 0\end{bmatrix}$

$\xrightarrow[\text{maple}]{\text{rref}}$ $\begin{array}{ccccc}\begin{bmatrix}1 & 0 & -2 & 0 & 0\\0 & 1 & 1 & 0 & 0\\0 & 0 & 0 & 1 & 0\end{bmatrix}\end{array}$

$\begin{array}{cccc}x_1 & x_2 & x_3 & x_4\\L & L & F & L\end{array}$

$x_3 = t \quad x_1 = +2x_3 = 2t$

$x_2 = -x_3 = -t$

$x_4 = 0$

$\boxed{\vec{x} = \begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} = \begin{bmatrix}2t\\-t\\t\\0\end{bmatrix}}$ b) $= t\begin{bmatrix}2\\-1\\1\\0\end{bmatrix}$

1-d soln space, basis vector: $\boxed{\langle 2,-1,1,0\rangle}$

② c) $2\vec{v}_1 - \vec{v}_2 + \vec{v}_3 = 0$

$\underbrace{2\begin{bmatrix}2\\1\\-1\end{bmatrix} - \begin{bmatrix}1\\2\\-3\end{bmatrix} + \begin{bmatrix}-3\\0\\-1\end{bmatrix}} = \begin{bmatrix}0\\0\\0\end{bmatrix}$

check: $\begin{bmatrix}4-1-3\\2-2+0\\-2+3-1\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$ ✓

$\vec{v}_3 = -2\vec{v}_1 + \vec{v}_2$

$\{\vec{v}_1, \vec{v}_2, \vec{v}_4\}$ remain and are 3 linearly independent vectors in $\mathbb{R}^3$ so they span $\mathbb{R}^3$ and are therefore a basis.

③ a) can we find coefficients $\langle x_1, x_2, x_3\rangle$?

$\begin{bmatrix}2 & 1 & 3\\1 & 2 & 4\\2 & 3 & 1\\1 & 4 & 2\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}6\\4\\2\\0\end{bmatrix}$

$\begin{bmatrix}2 & 1 & 3 & 6\\1 & 2 & 4 & 4\\2 & 3 & 1 & 2\\1 & 4 & 2 & 0\end{bmatrix} \xrightarrow[\text{maple}]{\text{rref}} \begin{bmatrix}1 & 0 & 0 & 2\\0 & 1 & 0 & -1\\0 & 0 & 1 & 1\\0 & 0 & 0 & 0\end{bmatrix} \rightarrow \begin{array}{l}x_1 = 2\\x_2 = -1\\x_3 = 1\end{array}$

$\begin{array}{ccc}x_1 & x_2 & x_3\\L & L & L\end{array}$

yes, we can!

so:

$\boxed{\begin{bmatrix}6\\4\\2\\6\end{bmatrix} = 2\begin{bmatrix}2\\1\\2\\1\end{bmatrix} - \begin{bmatrix}1\\2\\3\\4\end{bmatrix} + \begin{bmatrix}3\\4\\1\\2\end{bmatrix}} = \begin{bmatrix}4-1+3\\2-2+4\\4-3+1\\2-4+2\end{bmatrix} = \begin{bmatrix}6\\4\\2\\0\end{bmatrix}$ ✓

① a)