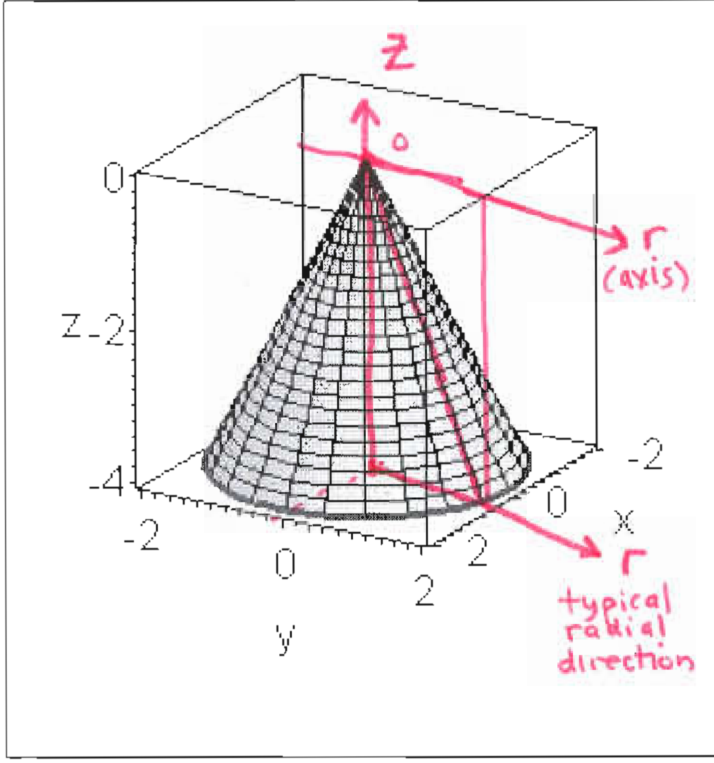
MAT2500-02 08S Quiz 9   Print Name (Last, First) ______________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers *exact* (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC).



1. The solid cone region R lies below the cone $z = -2\sqrt{x^2+y^2}$ and above the plane $z = -4$.

a) What is the equation describing the intersection of these two surfaces as expressed in cylindrical coordinates?

b) In order to iterate $\iiint_R f(x, y, z)\, dV$ as a triple integral in cylindrical coordinates, make a diagram of the *rz* half plane describing this solid, shading in the region of integration, and showing a typical vertical cross-section with its directional arrow indicating the inner integration along *z*, labeling its endpoints properly, and indicating the radial limits of the region. [Label axes, tickmarks, intercepts, etc.]

d) Using your diagram, write down the iterated integral representing $\iiint_R f(x, y, z)\, dV$ in cylindrical coordinates for any function $f(x, y, z) = F(r, \theta, z)$ when re-expressed in ~~spherical~~ cylindrical coordinates.

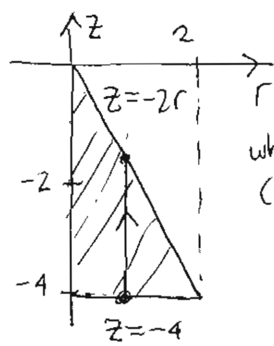
e) Now write down the corresponding iterated integrals $V = \iiint_R 1\, dV$ and $M_{xy} = \iiint_R z\, dV$.

f) Evaluate these two integrals by hand. Does your result lead to the correct average *z*-coordinate value $\bar{z} = M_{xy}/V = -\cancel{2}\,3$ (exactly!)? oops!

solution

a) $z = -2\sqrt{x^2+y^2} = -2r$, $z = -4$ $\Big\}\rightarrow \boxed{r=2}$ (in the plane $z=-4$)

b)



while $r = 0..2$ (and $\theta = 0..2\pi$)

c) $\iiint_R f(xyz)\, dV = \int_0^{2\pi}\int_0^2\int_{-4}^{-2r} F(r,\theta,z)\, r\, dz\, dr\, d\theta$

e) f) $V = \int_0^{2\pi}\int_0^2 \underbrace{\int_{-4}^{-2r} r\, dz}\, dr\, d\theta = \int_0^{2\pi} \frac{8}{3}\, d\theta = \frac{8}{3}(2\pi) = \boxed{\frac{16\pi}{3}}$

$zr\Big|_{z=-4}^{z=-2r} = r(-2r+4) = -2r^2 + 4r$

$\int_0^2 -2r^2 + 4r\, dr = -\frac{2}{3}r^3 + 2r^2\Big|_{r=0}^{r=2} = -\frac{2}{3}\cdot 8 + 2(4) = \boxed{8/3}$

$M_{xy} = \int_0^{2\pi}\int_0^2 \underbrace{\int_{-4}^{-2r} z\, r\, dz}\, dr\, d\theta = \int_0^{2\pi} -8\, d\theta = -8(2\pi) = \boxed{-16\pi}$

$\frac{rz^2}{2}\Big|_{z=-4}^{z=-2r} = \frac{r}{2}\left[4r^2 - 16\right]$

$\int_0^2 2r^3 - 8r\, dr = \frac{2r^4}{4} - \frac{8r^2}{2}\Big|_0^2 = \frac{2^4}{2} - 8\cdot 2 = -8$

$\bar{z} = \frac{M_{xy}}{V} = \frac{-16\pi}{16\pi/3} = \boxed{-3}$ yes!