MAT2500-02 08S Quiz 8 Print Name (Last, First) _____

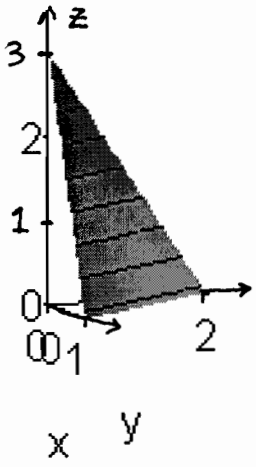
Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate) . Indicate where technology is used and what type (Maple, GC).

1. Consider the integral $\int_0^1 \int_x^{x^{\frac{1}{3}}} 4\,x\,y\,dy\,dx = 1/4$.

a) Make a diagram in the plane shading in the region of integration, and showing a typical cross-section with its directional arrow indicating the inner integration, labeling its endpoints properly. [Label axes, tickmarks, intercepts, etc.]

b) Now make a similar new diagram indicating the corresponding situation for the reversed order of integration, showing the typical cross-section with properly labeled endpoints.

c) Write down the new iterated double integral and evaluate it using technology (state which technology is used). Do you get the correct result? If not, can you track down your error in setup?



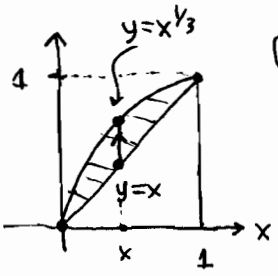
2. Consider the integral $\int_0^1 \int_0^{2-2x} \int_0^{3-3x-\frac{3}{2}y} 5\,x\,y\,dz\,dy\,dx = \frac{1}{2}$ over the region shown in the figure as the portion of the first octant below the plane.

a) Rewrite the integral in the order $\iiint \ldots dx\,dz\,dy$ first showing a plane diagram illustrating the new outer double integral as in problem 1 and indicating the starting and stopping values of x for the innermost integral.

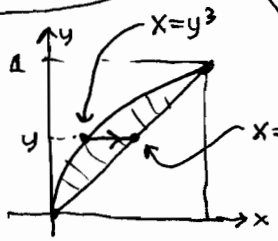
b) Evaluate your new integral using technology. Does it have the correct value? If not, can you track down your error in setup?

▶ solution

(1) a) $\int_{x=0}^{1} \int_{y=x}^{y=x^{1/3}} \ldots dy\,dx$



"as x = 0..1"



"as y = 0..1"

$y = x^{1/3} \rightarrow y^3 = x$

maple

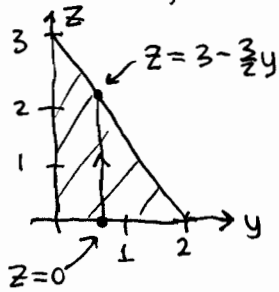
c) $\int_0^1 \int_{y^3}^{y} 4xy\,dx\,dy = \cdots = \frac{1}{4}$ ✓

(2) a) $\int_{x=0}^{1} \int_{y=0}^{2-2x} \int_{z=0}^{3-3x-\frac{3}{2}y} \ldots dz\,dy\,dx$

ceiling: $z = 3-3x-\frac{3}{2}y$

or: $x = (3-z-\frac{3}{2}y)/3 = 1-\frac{1}{2}y-\frac{1}{3}z$ ←

x first from back wall $x=0$ to front wall



front wall intersects $x=0$ at: $z = 3-0-\frac{3}{2}y$

(innermost integral: $x = 0 .. 1-\frac{1}{2}y-\frac{1}{3}z$)

as y = 0..2

so $\int_0^2 \int_0^{3-\frac{3}{2}y} \int_0^{1-\frac{1}{2}y-\frac{1}{3}z} 5xy\,dx\,dz\,dy = \cdots = \frac{1}{2}$

maple b)