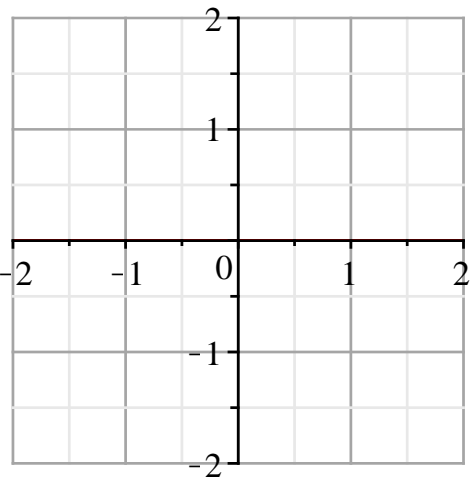Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC, MathCad). You are encouraged to use technology to check all of your hand results.

1. $f(x, y) = x^3 - 3xy + y^3$, $P(1, 0)$, $Q(-1, 1)$.
a) Verify that $(0, 0)$ and $(1, 1)$ are critical points of this function.
b) Use the second derivative test to classify these critical points as local minima, local maxima or saddle points.
c) Find the direction of maximum increase at P (direction specified by a unit vector!) and the directional derivative in that direction.
d) Evaluate the directional derivative at P in the direction of Q, using proper identifying symbols.
e) Make a fully labeled rough diagram of the two points and the two unit vectors associated with the directions of the previous two parts, and include the tangent line to the level curve of f through P.
f) Obtain the linear approximation $L(x, y)$ to f at P and use it to approximate $f(1.01, .05)$.
g) Use the chain rule to evaluate the derivative of f along the curve $\vec{r}(t) = \langle 1 - t, t \rangle$ at $t = 0$.

2. a) Write the equation of the tangent plane to the level surface of the function $g(x, y, z) = xy + yz + xz$ at $(1, 1, 1)$ and simplify it to the usual standard form. What is the equation of that level surface? Of the two unit vectors perpendicular to the tangent plane at that point, what are the components of the upwards pointing unit vector (z-component positive)?
b) What is the largest possible value of $xy + yz + xz$ for three positive numbers x, y, z whose sum is 1? [Find the relevant critical point and confirm it is a local max.]

▶ **solution**

▼ **pledge**

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet stapled on top of your answer sheets as a cover page, with the first test page facing up:
"During this examination, all work has been my own. I have not accessed any of the class web pages or any other sites during the exam. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: Date: