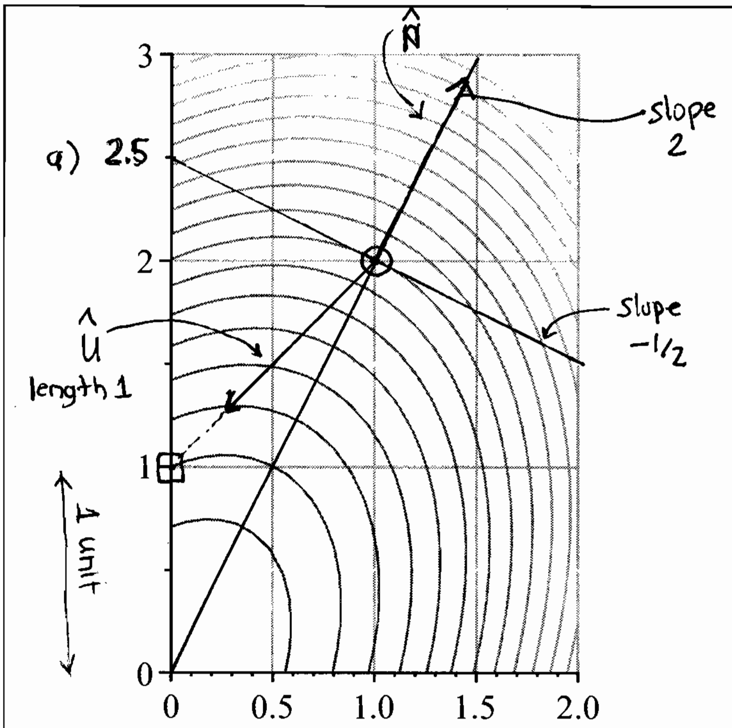
MAT2500-02 08S Quiz 6 Print Name (Last, First) ______________________________

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of each problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC).



1. $f(x,y) = 4x^2 - 2xy + \frac{5}{2}y^2$

a) Put a circled dot at the point (1,2) and draw in your guess of the tangent line and normal line to the level curve of the function through that point, using a straight edge.

b) Evaluate the gradient vector field and its value at (1,2), identifying it with its proper symbol, and draw its direction unit vector into your diagram.

c) Write an equation for the normal line to the level curve through (1,2). What is its y intercept? Does it seem consistent with your hand drawn line?

d) Write an equation for the tangent line to this level curve. What is its y intercept? Does it seem consistent with your hand drawn line?

e) What is the derivative of f at (1,2) in the direction of the point (0,1)? Identify it by its proper symbol and include the associated direction unit vector $\vec{u}$ in the diagram. Is the function increasing or decreasing in this direction?

f) Write an equation for the level curve at (1,2).

▶ solution

b) $f(x,y) = 4x^2 - 2xy + \frac{5}{2}y^2$

$f_x(x,y) = \frac{\partial}{\partial x}(4x^2 - 2xy + \frac{5}{2}y^2) = 8x - 2y + 0$

$f_y(x,y) = \frac{\partial}{\partial y}(4x^2 - 2xy + \frac{5}{2}y^2) = 0 - 2x + 5y$

$\boxed{\vec{\nabla} f(x,y) = \langle 8x - 2y, -2x + 5y\rangle}$

$\vec{N} = \boxed{\vec{\nabla} f(1,2) = \langle 8(1) - 2(2), -2(1) + 5(2)\rangle = \langle 8-4, -2+10\rangle = \langle 4, 8\rangle} = 4\langle 1,2\rangle \to$ slope 2 yes exactly my line. consistent (of course I cheated)

$|\vec{\nabla} f(1,2)| = 4\sqrt{1+4} = 4\sqrt{5}$

$\widehat{\nabla f}(1,2) = \frac{\langle 1,2\rangle}{\sqrt{5}} = \hat{N}$

c) slope 2, thru pt (1,2): $\frac{y-2}{x-1} = 2$

$y - 2 = 2(x-1) \to \boxed{y = 2 + 2(x-1) = 2x}$

$\boxed{y \text{ intercept is } 0} \to$ consistent (I cheated of course)

$\vec{r} = \vec{r}_0 + t\vec{N}$, $\langle x,y\rangle = \langle 1,2\rangle + t\langle 4,8\rangle = \langle 1+4t, 2+8t\rangle$

y intercept: $x = 0 \to t = -\frac{1}{4} \to y = 2 - 2 = 0$

$\vec{N}\cdot(\vec{r} - \vec{r}_0) = 0$: $4\langle 1,2\rangle\cdot(\langle x,y\rangle - \langle 1,2\rangle) = 0$

$(x-1) + 2(y-2) = 0 \to \boxed{x + 2y = 5} \to y = \frac{5}{2} - \frac{x}{2}$

d) perp line has negative reciprocal slope $-1/2$:

$\frac{y-2}{x-1} = -\frac{1}{2} \to y - 2 = -\frac{1}{2}(x-1)$

$\boxed{y = 2 - \frac{1}{2}x + \frac{1}{2} = \frac{5}{2} - \frac{x}{2}}$

yup! $\boxed{y\text{-intercept: } 2.5}$

e) $\vec{u} = \langle 0,1\rangle - \langle 1,2\rangle = \langle -1,-1\rangle$

$\hat{u} = -\frac{\langle 1,1\rangle}{\sqrt{2}}$

$D_{\hat{u}} f(1,2) = \hat{u}\cdot\vec{\nabla} f(1,2)$

$= -\frac{\langle 1,1\rangle}{\sqrt{2}}\cdot 4\langle 1,2\rangle = -2\sqrt{2}(1+2)$

$= \boxed{-6\sqrt{2}} < 0$

negative derivative $\to$ $\boxed{\text{decreasing}}$

f) $f(1,2) = 4(1)^2 - 2(1)(2) + \frac{5}{2}(2)^2$

$= 4 - 4 + 10 = 10$ so

$f(x,y) = f(1,2)$:

$\boxed{4x^2 - 2xy + \frac{5}{2}y^2 = 10}$