MAT2500-02 08S Quiz 5 Print Name (Last, First) ____________________

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate) . Indicate where technology is used and what type (Maple, GC).

1. $f(x, y) = x^2 + xy + 3y^2$

a) Find the linear approximation to $f$ at $(1, 1)$ and then evaluate that function $L(x, y)$ at the point $(0.98, 1.01)$. What is the value of the error: $E = f(0.98, 1.01) - L(0.98, 1.01)$?

b) Using your result for part a), write the equation of the tangent plane to $f$ at (1,1) and simplify it to the form $ax + by + cz = d$.

2. Given the table of values of the heat index function I (degrees F) of the temperature $T$ (degrees F) and relative humidity $H$ ( in percent), evaluate the two partial derivatives and then the linear approximation to $I$ at ( 94°F, 80%) and use it to estimate $I$ at (95°F, 78%).

| T\H | 75 | 80 | 85 |
|---|---|---|---|
| 92 | | 119 | |
| 94 | 122 | 127 | 132 |
| 96 | | 135 | |

## ▶ solution

① a) $f(x,y) = x^2+xy+3y^2$

$f_x(x,y) = \frac{\partial}{\partial x}(x^2+xy+3y^2) = 2x+y+0$

$f_y(x,y) = \frac{\partial}{\partial y}(x^2+xy+3y^2) = 0+x+6y$

$f_x(1,1) = 2+1 = 3$

$f_y(1,1) = 1+6 = 7$

$f(1,1) = 1+1+3 = 5$

$\boxed{L(x,y)} = f(1,1) + f_x(1,1)(x-1) + f_y(1,1)(y-1)$

$= 5+3(x-1)+7(y-1)$ $\quad (=3x+7y-5)$

$L(0.98, 1.01) = 5 + 3(\underbrace{0.98-1}_{-0.02}) + 7(\underbrace{1.01-1}_{.01})$

$= 5 - 0.06 + 0.07 = \boxed{5.01}$

$f(0.98, 1.01) = 5.0105$

$\boxed{E =} f(0.98,1.01) - L(0.98,1.01) = 5.0105 - 5.01$

$= 0.0005$

b) $z = L(x,y) = 5+3(x-1)+7(y-1)$

$= 5+3x-3+7y-7$

$\boxed{3x+7y-z=5}$

② 

$\Delta I = 8 \quad \Delta I = 8$

I: 119 127 135

T: 92 94 96

$\Delta T = 2 \quad \Delta T = 2$

$\frac{\Delta I}{\Delta T} = \frac{8}{2} = 4 \quad \frac{\Delta I}{\Delta T} = \frac{8}{2} = 4$

$\therefore \frac{\partial I}{\partial T}(94,80) = \frac{1}{2}(4+4) = 4$

$\Delta I = 5 \quad \Delta I = 5$

122 127 132

75 80 85

$\Delta H = 5 \quad \Delta H = 5$

$\frac{\Delta I}{\Delta T} = \frac{5}{5} = 1 \quad \frac{\Delta H}{\Delta T} = \frac{5}{5} = 1$

$\therefore \frac{\partial T}{\partial H}(94,80) = \frac{1}{2}(1+1) = 1$

$L(T,H) = I(94,80) + \frac{\partial I}{\partial T}(94,80)(T-94) + \frac{\partial I}{\partial H}(94,80)(H-80)$

$= 127 + 4(T-94) + 1(H-80)$ ← more useful NOT multiplied out!!

$L(95,78) = 127 + 4(95-94) + (78-80)$

$= 127 + 4 - 2 = \boxed{129\,°F}$