MAT2500-02 08s Quiz 4 Print Name (Last, First) ____________________________

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC).

1. $f(x,y) = y^2\cos(\pi x y)$ ~~[Note: all values for parts a) and b) are integers.]~~ oops

a) Evaluate f and the first partial derivatives of f at $(x,y) = (2,1)$ using proper notation for all derivatives evaluated in the process.

b) Evaluate the four second partial derivatives of f at $(x,y) = (2,1)$ using proper notation for all derivatives evaluated in the process.

c) Do your formulas for the two mixed partial derivative functions f_{xy} and f_{yx} agree as they should?

d) What can you conclude about how f is increasing or decreasing at (2,1) as you increase x and y respectively? What can you conclude about the concavity of the cross-sectional curves for $y=1$ and $x=2$ respectively?

▶ solution

a) $f(x,y) = y^2\cos(\pi x y)$

each step shown

$$f_x(x,y) = \frac{\partial}{\partial x} y^2\cos(\pi xy) = y^2 \frac{\partial}{\partial x}\cos(\pi xy) = y^2(-\sin(\pi xy))\underbrace{\frac{\partial}{\partial x}(\pi xy)}_{\pi y} = \boxed{-\pi y^3 \sin(\pi xy)}$$

$$f_y(x,y) = \frac{\partial}{\partial y}(y^2\cos(\pi xy)) = \underbrace{\left(\frac{\partial}{\partial y}y^2\right)}_{2y}\cos(\pi xy) + y^2\underbrace{\frac{\partial}{\partial y}\cos(\pi xy)}_{-\sin \pi xy \underbrace{\frac{\partial}{\partial y}(\pi xy)}_{\pi x}} = \boxed{2y\cos(\pi xy) - \pi x y^2 \sin(\pi xy)}$$

$$\boxed{\begin{aligned} f(2,1) &= 1^2\cos(2\pi) = 1 \\ f_x(2,1) &= -\pi 1^2 \sin(2\pi) = 0 \\ f_y(2,1) &= 2(1)\cos(2\pi) - \pi(1)(2)\sin 2\pi = 2 \end{aligned}}$$

d)

$f_x(2,1) = 0$ ← not increasing or decreasing

$f_y(2,1) = 2 > 0$ ← increasing

now I dont show each step by itself:

b) $$\boxed{f_{xy}(x,y)} = \frac{\partial}{\partial y} f_x(x,y) = \frac{\partial}{\partial y}(-\pi y^3 \sin \pi xy) = -\pi[3y^2 \sin \pi xy + y^3\cos(\pi xy)(\pi x)]$$
$$= -\pi[3y^2\sin(\pi xy) + \pi x y^3 \cos(\pi xy)]$$

$$\boxed{f_{yx}(x,y)} = \frac{\partial}{\partial x} f_y(x,y) = \frac{\partial}{\partial x}[2y\cos(\pi xy) - \pi x y^2 \sin(\pi xy)] = 2y(-\sin(\pi xy))(\pi y) - \pi y^2(1 \sin(\pi xy) + x\cos(\pi xy)(\pi y))$$
$$= -\pi(3y^2\sin(\pi xy) + \pi x y^3\cos(\pi xy)) = f_{xy}(x,y)$$

$$f_{xx}(x,y) = \frac{\partial}{\partial x} f_x(x,y) = \frac{\partial}{\partial x}(-\pi y^3 \sin \pi xy) = -\pi y^3 \cos(\pi xy)(\pi y) = \boxed{-\pi^2 y^4 \cos \pi xy}$$

$$\boxed{f_{yy}(x,y)} = \frac{\partial}{\partial y} f_y(x,y) = \frac{\partial}{\partial y}(2y\cos \pi xy - \pi x y^2 \sin(\pi xy)) = 2\cos \pi xy + 2y\sin(\pi xy)(\pi x) - 2\pi xy \sin \pi xy - \pi x y^2 \cos(\pi xy)(\pi x)$$
$$= (2 - \pi^2 x^2 y^2)\cos(\pi xy)$$

$$\boxed{\begin{aligned} f_{xx}(2,1) &= -\pi^2 \cos 2\pi = -\pi^2 < 0 \\ f_{yy}(2,1) &= (2 - 4\pi^2)\cos 2\pi = 2 - 4\pi^2 < 0 \\ f_{xy}(2,1) &= f_{yx}(2,1) = -\pi(3\sin 2\pi + 2\pi\cos 2\pi) \\ &= -2\pi^2 \end{aligned}}$$

d) both concave down