MAT2500-02 08S Quiz 3 Print Name (Last, First) ________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

Given the vector-valued function $\vec{r}(t) = \langle t, t^2, t^3\rangle$ for its natural domain $-\infty < t < \infty$:

a) Evaluate $\vec{r}\,'(t)$, $\vec{r}\,''(t)$, $|\vec{r}\,'(t)|$, $\vec{T}(t)$ and remember to simplify your results (no credit for unidentified expressions).

b) Evaluate $\vec{r}(1)$, $\vec{r}\,'(1)$, $\vec{r}\,''(1)$, $\vec{T}(1)$ and remember to simplify your results (no credit for unidentified expressions).

c) Evaluate the exact angle θ in radians between $\vec{r}\,'(1)$ and $\vec{r}\,''(1)$ and a single decimal place approximation in degrees.

d) Evaluate the vector $\vec{a}$ which is the vector projection of $\vec{r}\,''(1)$ orthogonal to $\vec{r}\,'(1)$.

e) **Optional:** Make a suggestive hand diagram illustrating the various vectors derived above at $\vec{r}(1)$.

▶ solution

a)
$$\boxed{\begin{aligned}\vec{r} &= \langle t, t^2, t^3\rangle\\ \vec{r}\,' &= \langle 1, 2t, 3t^2\rangle\\ \vec{r}\,'' &= \langle 0, 2, 6t\rangle\end{aligned}} \to \boxed{|\vec{r}\,'| = \sqrt{1+4t^2+9t^4}} \to \hat{T} = \frac{\vec{r}\,'}{|r'|} = \boxed{\frac{\langle 1, 2t, 3t^2\rangle}{\sqrt{1+4t^2+9t^4}}}$$

b)
$$\boxed{\begin{aligned}\vec{r}(1) &= \langle 1, 1, 1\rangle\\ \vec{r}\,'(1) &= \langle 1, 2, 3\rangle\\ \vec{r}\,''(1) &= \langle 0, 2, 6\rangle\\ \hat{T}(1) &= \frac{\langle 1, 2, 3\rangle}{\sqrt{14}}\end{aligned}} \qquad \boxed{|r'(1)| = \sqrt{1+4+9} = \sqrt{14}} \approx 3.74$$

c) $\vec{r}\,''(1) = \langle 0, 2, 6\rangle = 2\langle 0, 1, 3\rangle$

$$\widehat{r''(1)} = \frac{\langle 0, 1, 3\rangle}{\sqrt{1+9}} = \frac{\langle 0, 1, 3\rangle}{\sqrt{10}}$$

$$\cos\theta = \hat{T}(1)\cdot\widehat{r''(1)} = \frac{\langle 1, 2, 3\rangle}{\sqrt{14}}\cdot\frac{\langle 0, 1, 3\rangle}{\sqrt{10}}$$

$$= \frac{1(0)+2(1)+3(3)}{\sqrt{2\cdot 7}\sqrt{2\cdot 5}} = \frac{11}{\sqrt{14}\sqrt{10}} = \frac{11}{2\sqrt{35}}$$

$$\boxed{\theta = \arccos\left(\frac{11}{\sqrt{14}\sqrt{10}}\right) \approx 21.6^\circ} \leftarrow \text{maple}$$

d) $\vec{r}\,''(1) = \langle 0, 2, 6\rangle$

scalar comp
$$\hat{T}(1)\cdot\vec{r}\,''(1) = \frac{\langle 1, 2, 3\rangle\cdot\langle 0, 2, 6\rangle}{\sqrt{14}} = \frac{0+4+18}{\sqrt{14}} = \frac{22}{\sqrt{14}}$$

vector comp along $\hat{T}(1)$
$$\left(\hat{T}(1)\cdot\vec{r}\,''(1)\right)\hat{T}(1) = \frac{22}{\sqrt{14}}\,\frac{\langle 1, 2, 3\rangle}{\sqrt{14}} = \frac{22}{14}\langle 1, 2, 3\rangle = \frac{11}{7}\langle 1, 2, 3\rangle$$
$$= [\vec{r}\,''(1)]_\parallel$$

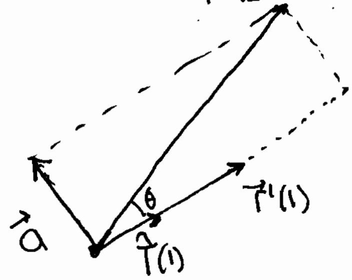
d) continued:

$$[\vec{r}\,''(1)]_\perp = \vec{r}\,''(1) - [r''(1)]_\parallel$$
$$= \langle 0, 2, 6\rangle - \left\langle \tfrac{11}{7}, \tfrac{22}{7}, \tfrac{33}{7}\right\rangle$$
$$= \left\langle -\tfrac{11}{7}, \tfrac{14-22}{7}, \tfrac{42-33}{7}\right\rangle$$
$$\boxed{\begin{aligned}&= \left\langle -\tfrac{11}{7}, -\tfrac{8}{7}, \tfrac{9}{7}\right\rangle\\ &= \tfrac{1}{7}\langle -11, -8, 9\rangle = \vec{a}\end{aligned}}$$

e) $\|\vec{r}\,''(1)\| = 2|\langle 0, 1, 3\rangle| = 2\sqrt{10} \approx 6.32$

$|\vec{a}| = \dfrac{\sqrt{121+64+81}}{7} = \dfrac{\sqrt{266}}{7} \approx 2.33$

Knowing the lengths & relative angle we can make the diagram



It is too challenging to do this in 3D attached to the curve, so we can just make the diagram in the plane of r′ and r″!