MAT2500-02 08S Quiz 1 Print Name (Last, First) ________________________________

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

Given three points $P1(1,2,3)$, $P2(1,3,2)$, $P3(0,1,1)$:

a) Evaluate the lengths of the two difference vectors $\overrightarrow{\mathbf{P_1P_2}}$ and $\overrightarrow{\mathbf{P_1P_3}}$ exactly and numerically to 2 decimal place accuracy.

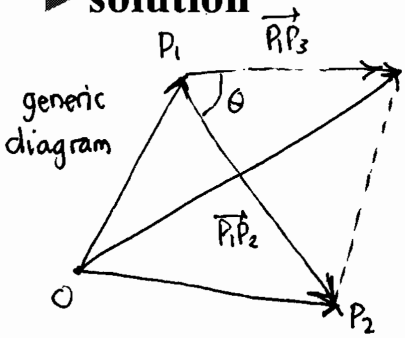
b) Find the angle between $\overrightarrow{\mathbf{P_1P_2}}$ and $\overrightarrow{\mathbf{P_1P_3}}$ exactly in radians (as an inverse trig expression) and numerically in degrees to 1 decimal place accuracy.

c) Find the vector component $\overrightarrow{\mathbf{P_1P_2}}_{\parallel}$ of $\overrightarrow{\mathbf{P_1P_2}}$ along $\overrightarrow{\mathbf{P_1P_3}}$ and the vector component $\overrightarrow{\mathbf{P_1P_2}}_{\perp}$ of $\overrightarrow{\mathbf{P_1P_2}}$ perpendicular to $\overrightarrow{\mathbf{P_1P_3}}$ both exactly and numerically to 2 decimal place accuracy. [Is your last result perpendicular to the direction along which we are projecting?]

d) Evaluate the length of this vector component perpendicular to $\overrightarrow{\mathbf{P_1P_3}}$, both exactly and to 2 decimal place accuracy.

e) Knowing the lengths of two sides of the triangle and the angle between them from parts a) and b), make a rough sketch of the triangle itself independent of its position in space and include the rectangle associated with the projection/decomposition process you evaluated and label everything in the sketch (vertices are points, label three sides with difference vectors and put in arrows to indicate their direction, etc.). Does your result for part d) seem consistent with your drawing? Explain.

▶ **solution**



a) $\overrightarrow{OP_1} = \langle 1,2,3\rangle$ $\quad \overrightarrow{P_1P_2} = \overrightarrow{OP_2} - \overrightarrow{OP_1} = \langle 1,3,2\rangle - \langle 1,2,3\rangle = \langle 0,1,-1\rangle$

$\overrightarrow{OP_2} = \langle 1,3,2\rangle$ $\quad \overrightarrow{P_1P_3} = \overrightarrow{OP_3} - \overrightarrow{OP_1} = \langle 0,1,1\rangle - \langle 1,2,3\rangle = \langle -1,-1,-2\rangle$

$\overrightarrow{OP_3} = \langle 0,1,1\rangle$

$|\overrightarrow{P_1P_2}| = \sqrt{1+1} = \boxed{\sqrt{2}} \approx \boxed{1.41}$

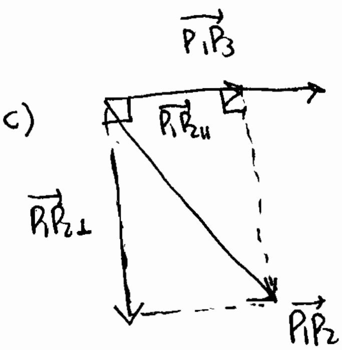
$|\overrightarrow{P_1P_3}| = \sqrt{1+1+4} = \boxed{\sqrt{6}} \approx \boxed{2.45}$

b) $\widehat{P_1P_2} = \overrightarrow{P_1P_2}/|\overrightarrow{P_1P_2}| = \dfrac{\langle 0,1,-1\rangle}{\sqrt{2}}$ $\quad \cos\theta = \widehat{P_1P_2}\cdot\widehat{P_1P_3} = \dfrac{\langle 0,1,-1\rangle}{\sqrt{2}}\cdot\dfrac{\langle -1,-1,-2\rangle}{\sqrt{6}}$

$\widehat{P_1P_3} = \overrightarrow{P_1P_3}/|\overrightarrow{P_1P_3}| = \dfrac{\langle -1,-1,-2\rangle}{\sqrt{6}}$ $\quad = \dfrac{0-1+2}{\sqrt{12}} = \dfrac{1}{\sqrt{12}}$

$\theta = \boxed{\arccos \frac{1}{\sqrt{12}} \approx 73.2^\circ}$

c)



$\overrightarrow{P_1P_2}\cdot\widehat{P_1P_3} = \langle 0,1,-1\rangle\cdot\dfrac{\langle -1,-1,-2\rangle}{\sqrt{6}} = \dfrac{-1+2}{\sqrt{6}} = \dfrac{1}{\sqrt{6}}$ (scalar component) ≈ 0.41

$\overrightarrow{P_1P_2}_{\parallel} = (\overrightarrow{P_1P_2}\cdot\widehat{P_1P_3})(\widehat{P_1P_3}) = \dfrac{1}{\sqrt{6}}\dfrac{\langle -1,-1,-2\rangle}{\sqrt{6}} = \boxed{-\frac{1}{6}\langle 1,1,2\rangle} = \boxed{\langle -\frac{1}{6},-\frac{1}{6},-\frac{1}{3}\rangle}$

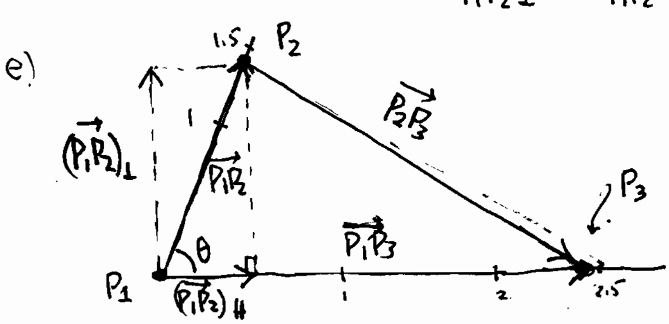
$\overrightarrow{P_1P_2}_{\perp} = \overrightarrow{P_1P_2} - \overrightarrow{P_1P_2}_{\parallel} = \langle 0,1,-1\rangle - (-\frac{1}{6})\langle 1,1,2\rangle$ oops!

$= \langle 0,1,-1\rangle + \langle \frac{1}{6},\frac{1}{6},\frac{2}{6}\rangle = \boxed{\langle \frac{1}{6},\frac{7}{6},-\frac{4}{6}\rangle = \frac{1}{6}\langle 1,7,-4\rangle}$

$|\overrightarrow{P_1P_2}_{\perp}| = \frac{1}{6}\sqrt{1+49+16} = \boxed{\frac{\sqrt{66}}{6} = \cancel{\ldots} \approx 1.35}$

oops!

e)



using $|\overrightarrow{P_1P_3}| \approx 2.45$, $|\overrightarrow{P_1P_2}| \approx 1.41$, $\theta \approx 73^\circ$ (guessing angle)

yes, in diagram: $\overrightarrow{P_1P_2}_{\perp}$ is a bit shorter than $\overrightarrow{P_1P_2}$!