MAT2500-02 08S TEST2 ANSWERS

① $f(x,y) = x^3 - 3xy + y^3$

$f_x(x,y) = \frac{\partial}{\partial x}(x^3-3xy+y^3) = 3x^2 - 3y + 0 = 3x^2-3y = 3(x^2-y) = 0$

$f_y(x,y) = \frac{\partial}{\partial y}(x^3-3xy+y^3) = 0 - 3x + 3y^2 = -3x+3y^2 = 3(-x+y^2) = 0$

TO DERIVE:

$\rightarrow y = x^2$

$\rightarrow -x + y^2 = 0 \rightarrow -x + (x^2)^2 = 0$

$x(x^3-1) = 0 \rightarrow x = 0$ or $x = 1$

$y = 0^2 = 0 \quad x = 1^2 = 1$

$\therefore (0,0), (1,1)$ are crit pts

$f_x(0,0) = 3(0^2-0) = 0 \qquad f_x(1,1) = 3(1^2-1) = 0$

$f_y(0,0) = 3(-0+0^2) = 0 \qquad f_y(1,1) = 3(-1+1^2) = 0$

$\therefore$ (0,0) and (1,1) are critical points for f

b) $f_{xx}(x,y) = \frac{\partial}{\partial x}(3x^2-3y) = 6x - 0 = 6x$

$f_{yy}(x,y) = \frac{\partial}{\partial y}(-3x+3y^2) = 0 + 6y = 6y$

$f_{xy}(x,y) = \frac{\partial}{\partial y}(3x^2-3y) = -3$

	(0,0)	(1,1)	
f_{xx}	0	$6>0$ ✓	local min?
f_{yy}	0	$6>0$ ✓	
f_{xy}	-3	-3	
$f_{xx}f_{yy}-f_{xy}^2$	$-9<0$	$36-9>0$ ✓	$\boxed{\text{confirmed local min}}$
	$\boxed{\text{Saddle}}$		

c) $\vec{\nabla} f(x,y) = 3\langle x^2-y, -x+y^2\rangle$

$\vec{\nabla} f(1,0) = 3\langle 1,-1\rangle$

$\underbrace{\hat{u} = \widehat{\nabla f(1,0)} = \frac{\langle 1,-1\rangle}{\sqrt{2}}}_{\text{direction of max increase}}, \quad \underbrace{|\vec{\nabla} f(1,0)| = 3\sqrt{2}}_{D_{\hat{u}} f(1,0)}$

directional derivative in that direction

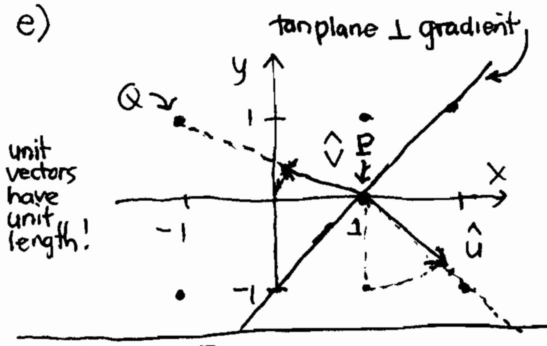
d) $\vec{PQ} = \langle -1,1\rangle - \langle 1,0\rangle = \langle -2,1\rangle$

$\hat{v} = \widehat{PQ} = \frac{\langle -2,1\rangle}{\sqrt{5}}$

$D_{\hat{v}} f(1,0) = \hat{v}\cdot\vec{\nabla} f(1,0) = \frac{\langle -2,1\rangle}{\sqrt{5}} \cdot 3\langle 1,-1\rangle$

$= \frac{3(-2-1)}{\sqrt{5}} = \boxed{-\frac{9}{\sqrt{5}}} \approx -4.025$ (decreasing)

e)



f) $f(1,0) = 1^3 - 3(1)(0) + 0^3 = 1$

$f_x(1,0) = 3(1^2-0) = 3$

$f_y(1,0) = 3(-1+0) = -3$

$$\boxed{L(x,y) = f(1,0) + f_x(1,0)(x-1) + f_y(1,0)(y-0) = 1 + 3(x-1) - 3(y)}$$

$f(1.01, 0.05) \approx L(1.01, 0.05) = 1 + 3(1.01-1) - 3(0.05)$

$= 1 + 3(.01) - 0.15 = 1 + .03 - .15$

$= 1 - 0.12 = \boxed{0.88}$

CHAIN RULE:

g) $\frac{d}{dt} f(\vec{r}(t)) = \frac{\partial f}{\partial x}(\vec{r}(t))\frac{dx}{dt} + \frac{\partial f}{\partial y}(\vec{r}(t))\frac{dy}{dt}$

$x = 1-t \quad \frac{dx}{dt} = -1$

$y = +t \quad \frac{dy}{dt} = 1$

$= [3(1-t)^2 - 3t](-1) + [-3(1-t) + 3t^2]$

$= \underbrace{-3(1-t)^2}_{-3(1-2t+t^2)} + 3t - 3 + 3t + 3t^2$

$= -3 + 6t - 3t^2 - 3 + 3t + 3t^2 = -6 + 9t$

$f_x(1-t, t) = 3(1-t)^2 - 3(t)$

$f_y(1-t, t) = -3(1-t) + 3(t)^2$

$\boxed{-6}$

$t = 0 \quad t=0$

② a) $g(x,y,z) = xy + yz + xz$, $\vec{r}_0 = \langle 1,1,1\rangle$, $g(1,1,1) = 3$

$g_x(x,y,z) = y+z$

$g_y(x,y,z) = x+z$

$g_z(x,y,z) = y+x$

$\vec{\nabla} g(1,1,1) = \langle 1+1, 1+1, 1+1\rangle = 2\underbrace{\langle 1,1,1\rangle}_{\vec{n}}$

$0 = \vec{n}\cdot(\vec{r}-\vec{r}_0) = \langle 1,1,1\rangle \cdot \langle x-1, y-1, z-1\rangle = x-1+y-1+z-1$

$\boxed{x+y+z=3}$ tan plane to level surface: $\boxed{xy+yz+xz=3}$

$\pm\hat{n} = \frac{\langle 1,1,1\rangle}{\sqrt{3}}$ ← unit normals to surfaces; choose + sign for upward normal

b) $x+y+z = 1 \rightarrow z = 1-x-y$

$f(x,y) = g(x,y,1-x-y) = xy + (x+y)(1-x-y)$

$= xy + x + y - (x+y)^2$

$f_x = y + 1 - 2(x+y) = -2x - y + 1 = 0$

$f_y = x + 1 - 2(x+y) = -x - 2y + 1 = 0$

$\rightarrow y = 1-2x: \quad x - 2(1-2x) + 1 = 0$

$-x - 2 + 4x + 1 = 0$

$3x = 1 \rightarrow x = 1/3 \rightarrow y = 1 - \frac{2}{3} = \frac{1}{3}$

$\rightarrow z = 1 - 1/3 - 1/3 = 1/3$

$g(1/3, 1/3, 1/3) = 1/9 + 1/9 + 1/9 = 3/9 = \boxed{1/3}$ FINAL ANSWER

$f_{xx} = -2, f_{yy} = -2$ <0 <0 local max? $f_{xy} = -1 \rightarrow f_{xx}f_{yy} - f_{xy}^2 = 4 - (-1)^2 > 0$ ✓ YES