MAT 2500-02 08F TEST 1 Answers

a) $\vec{r}(t) = \langle t, t^2, 1-t^2 \rangle$

$\vec{r}\,'(t) = \langle 1, 2t, -2t \rangle$

$\vec{r}\,''(t) = \langle 0, 2, -2 \rangle$

$|\vec{r}\,'(t)| = \sqrt{1+4t^2+4t^2} = \sqrt{1+8t^2}$

$\hat{T}(t) = \dfrac{\vec{r}\,'(t)}{|\vec{r}\,'(t)|} = \dfrac{\langle 1, 2t, -2t \rangle}{\sqrt{1+8t^2}}$

b) $\vec{r}(1) = \langle 1, 1, 1-1 \rangle = \langle 1, 1, 0 \rangle$

$\vec{r}\,'(1) = \langle 1, 2, -2 \rangle$

$\vec{r}\,''(1) = \langle 0, 2, -2 \rangle = \vec{a}(1)$

$\hat{T}(1) = \dfrac{\langle 1, 2, -2 \rangle}{\sqrt{1+8}} = \frac{1}{3}\langle 1, 2, -2 \rangle$

$\vec{r}\,'(1) \times \vec{r}\,''(1) = \begin{vmatrix} i & j & k \\ 1 & 2 & -2 \\ 0 & 2 & -2 \end{vmatrix}$!! careless

$= \langle 2(-2)-(-2)(2), (-2)(0) - \overbrace{(1)(2)}, 1(2)-2(0) \rangle$

$= \langle 0, 2, 2 \rangle = 2\underbrace{\langle 0, 1, 1 \rangle}_{\vec{n}}$

oops!

c) $\vec{r}_0 = \vec{r}(1) = \langle 1, 1, 0 \rangle$

$\vec{n} = \langle 0, 1, 1 \rangle$ orientation of acceleration plane at $t=1$

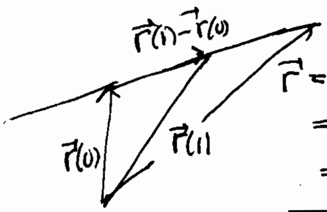
$\vec{n} \cdot (\vec{r} - \vec{r}_0) = 0$

$\langle 0, 1, 1 \rangle \cdot \langle x-1, y-1, z-0 \rangle = 0$

$(y-1) + z = 0 \quad \boxed{y+z=1}$

d) $\vec{r}(1) - \vec{r}(0) = \langle 1, 1, 0 \rangle - \langle 0, 0, 1 \rangle$

$= \langle 1, 1, -1 \rangle$



$\vec{r} = r(0) + t(\vec{r}(1) - \vec{r}(0))$

$= \langle 0, 0, 1 \rangle + t\langle 1, 1, -1 \rangle$

$= \langle t, t, 1-t \rangle = \langle x, y, z \rangle$

$\boxed{x = t,\ y = t,\ z = 1-t}$

(or $\vec{r} = \vec{r}(1) + t(\vec{r}(0) - \vec{r}(1))$

$= \langle 1, 1, 0 \rangle + t\langle -1, -1, 1 \rangle$

$= \langle 1-t, 1-t, t \rangle = \langle x, y, z \rangle$ etc)

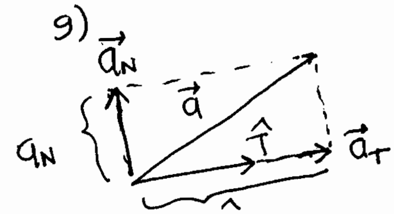
e) $|\vec{r}(0) - \vec{r}(1)| = |\langle 1, 1, -1 \rangle| = \sqrt{1+1+1} = \boxed{\sqrt{3}}$

$\underset{\text{maple}}{\approx} 1.73205 \approx \boxed{1.73}$

f) $L = \int_0^1 |\vec{r}\,'(t)|\,dt = \int_0^1 \sqrt{1+8t^2}\,dt$

$\overset{\text{maple}}{\approx} 1.8116 \approx \boxed{1.81}$ a bit longer than the straight line path between the endpoints, as it should be

g)



$a_T = a \cdot \hat{T}$

$a_T(1) = \hat{T}(1) \cdot \vec{a}(1) = \frac{1}{3}\langle 1, 2, -2 \rangle \cdot \langle 0, 2, -2 \rangle$

$= \frac{1}{3}(4+4) = \boxed{\frac{8}{3}} \overset{\cancel{\text{maple}}}{\underset{\text{bob!}}{\approx}} \boxed{2.67}$ (oops! 2.667)

$\vec{a}_T(1) = a_T(1)\hat{T}(1) = \frac{8}{3}\,\frac{1}{3}\langle 1, 2, -2 \rangle = \boxed{\frac{8}{9}\langle 1, 2, -2 \rangle}$

$\vec{a}_N(1) = \vec{a}(1) - \vec{a}_T(1) = \langle 0, 2, -2 \rangle - \langle \frac{8}{9}, \frac{16}{9}, -\frac{16}{9} \rangle$

$= \langle -\frac{8}{9}, \frac{18-16}{9}, \frac{-18+16}{9} \rangle = \frac{1}{9}\langle -8, 2, -2 \rangle$

$= \boxed{\frac{2}{9}\langle -4, 1, -1 \rangle}$ oops, almost mental error!

$a_N(1) = |\vec{a}_N(1)| = \frac{2}{9}|\langle -4, 1, -1 \rangle| = \frac{2}{9}\sqrt{16+1+1} = \boxed{\frac{2\sqrt{2}}{3}}$

or

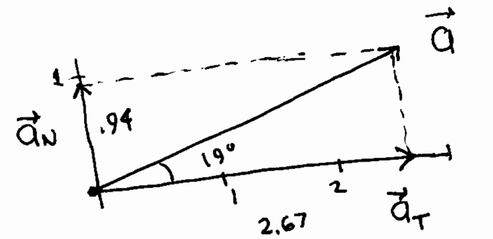
$a_N(1) = |\hat{T}(1) \times a(1)| = \dfrac{|\vec{r}\,'(1) \times \vec{r}\,''(1)|}{|\vec{r}\,'(1)|} = \dfrac{|\langle 0, 2, 2 \rangle|}{3}$

$= \frac{2}{3}|\langle 0, 1, 1 \rangle| = \boxed{\frac{2}{3}\sqrt{2}} \overset{\text{maple}}{\approx} \boxed{0.94}$ (oops! 0.934)

same! (check on errors)

h) $\theta = \arccos(\hat{T}(1) \cdot \hat{a}(1)) = \arccos\left(\dfrac{\langle 1, 2, -2 \rangle}{3} \cdot \dfrac{\langle 0, 1, -1 \rangle}{\sqrt{2}}\right)$

$= \arccos\left(\dfrac{2+2}{3\sqrt{2}}\right) = \arccos\left(\dfrac{4}{3\sqrt{2}}\right) \underset{\text{maple}}{\approx} 19.47° \approx \boxed{19°}$



this is the picture in the acceleration plane.