MAT2705-01 08F Quiz 5 Print Name (Last, First) ________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). **[You should use technology for row reductions and determinants.]**

1. $v_1 = \langle 1, 2, 3\rangle$, $v_2 = \langle 1, -1, 1\rangle$, $v_3 = \langle 2, 1, 4\rangle$, $v_4 = \langle 3, 3, 7\rangle$, $v_5 = \langle 0, 6, 4\rangle$

a) Express v_5 as a linear combination of the remaining 4 vectors, in the most general way. [Final answer: $v_5 = \ldots\, v_1 + \ldots$]

b) Check that this general linear combination that you find actually evaluates to v_5.

c) Find the independent linear relationships among these 4 vectors v_1, v_2, v_3, v_4 (i.e., what ***independent*** linear combinations of these vectors equal the zero vector?). Write out these relationships individually.

2. Demonstrate that the following vectors are linearly independent (explain your reasoning): $u_1 = \langle 1, 2, 3\rangle$, $u_2 = \langle 2, 3, 4\rangle$, $u_3 = \langle 3, 4, 1\rangle$.

▶ solution

1.a) $x_1\vec{v}_1 + x_2\vec{v}_2 + x_3\vec{v}_3 + x_4\vec{v}_4 \overset{?}{=} \vec{v}_5$

matrix form of vector equation:
$$x_1\begin{bmatrix}1\\2\\3\end{bmatrix} + x_2\begin{bmatrix}1\\-1\\1\end{bmatrix} + x_3\begin{bmatrix}2\\1\\4\end{bmatrix} + x_4\begin{bmatrix}3\\3\\7\end{bmatrix} = \begin{bmatrix}0\\6\\4\end{bmatrix} \to \begin{bmatrix}1&1&2&3\\2&-1&1&3\\3&1&4&7\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} = \begin{bmatrix}0\\6\\4\end{bmatrix}$$

augmented matrix:
$$\begin{bmatrix}1&1&2&3\\2&-1&1&3\\3&1&4&7\end{bmatrix} \xrightarrow{\text{Maple}} \begin{array}{c}\begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array}\\ \begin{bmatrix}1&0&1&2&2\\0&1&1&1&-2\\0&0&0&0&0\end{bmatrix}\\ \begin{array}{cccc} L & L & F & F \end{array}\end{array}$$

$x_1 + x_3 + 2x_4 = 2$
$x_2 + x_3 + x_4 = -2$ $\xrightarrow{\text{solve}}$ $x_1 = -t_1 - 2t_2$, $x_2 = -t_1 - t_2$

free: $\begin{cases} x_3 = t_1 \\ x_4 = t_2 \end{cases}$ Backsub

general solution:
$$\vec{X} = \begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} = \begin{bmatrix}2-t_1-2t_2\\-2-t_1-t_2\\t_1\\t_2\end{bmatrix} = t_1\begin{bmatrix}-1\\-1\\1\\0\end{bmatrix} + t_2\begin{bmatrix}-2\\-1\\0\\1\end{bmatrix} + \begin{bmatrix}2\\-2\\0\\0\end{bmatrix}$$

therefore $\boxed{\vec{v}_5 = (2-t_1-2t_2)\vec{v}_1 + (-2-t_1-t_2)\vec{v}_2 + t_1\vec{v}_3 + t_2\vec{v}_4}$

b)
$$(2-t_1-2t_2)\begin{bmatrix}1\\2\\3\end{bmatrix} + (-2-t_1-t_2)\begin{bmatrix}1\\-1\\1\end{bmatrix} + t_1\begin{bmatrix}2\\1\\4\end{bmatrix} + t_2\begin{bmatrix}3\\3\\7\end{bmatrix} = \begin{bmatrix}2-t_1-2t_2 + -2-t_1-t_2 + 2t_1+3t_2\\4-2t_1-4t_2 + 2+t_1+t_2 + t_1+3t_2\\6-3t_1-6t_2 - 2-t_1-t_2 + 4t_1+7t_2\end{bmatrix}$$
$$= \begin{bmatrix}0 + t_1(-1-1+2) + t_2(-2-1+3)\\4 + t_1(-2+1+1) + t_2(-4+1+3)\\6 + t_1(-3-1+4) + t_2(-6-1+7)\end{bmatrix} = \begin{bmatrix}0\\4\\6\end{bmatrix} \checkmark$$

c) The coefficients of t_1 and t_2 in $\vec{X}$ are the independent coefficient vectors:
$$\boxed{-\vec{v}_1 - \vec{v}_2 + \vec{v}_3 = 0, \quad -2\vec{v}_1 - \vec{v}_2 + \vec{v}_4 = 0}$$

2) $\begin{vmatrix}1&2&3\\2&3&4\\3&4&1\end{vmatrix} = 4 \neq 0$ so these vectors are linearly independent; $\begin{bmatrix}1&2&3\\2&3&4\\3&4&1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$ only has the zero solution

since the matrix of coefficients is invertible & row reduces to the identity matrix