MAT2705-01 08F Quiz 1 Print Name (Last, First) _____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

1. Write a differential equation that models the situation: "The acceleration $dv/dt$ of a Lambergini is proportional to the difference between 250 km/hr and the velocity of the car." If you use common sense, can you say something about the sign of the proportionality constant (i.e., should the acceleration be positive or negative when the speed is below 250?)?

2. $\frac{dy}{dx} = x - y, y(0) = 10, y(x) = Ce^{-x} + x - 1$

a) Verify that $y(x)$ satisfies the given differential equation.
b) Determine a value of the constant $C$ so that $y(x)$ satisfies the given initial condition.
c) Box your final result for $y(x)$. Then check that it indeed satisfies the initial condition.

Organize your work as though you were playing professor.

## ▶ solution

① $\frac{dv}{dt} \propto 250 - v \rightarrow \frac{dv}{dt} = k(250 - v)$

$\uparrow$ $\geq 0$ as v increases from 0 towards 250

$k > 0$ for positive acceleration as in moves forward picking up speed.

[or $\frac{dv}{dt} \propto v - 250 \rightarrow \frac{dv}{dt} = k_2(v - 250)$ in fact $k_2 = -k$

$\downarrow$ $\leq 0$ as v increases from 0

$k_2 < 0$ for positive acceleration]

② a) $y(x) = Ce^{-x} + x - 1$

$y'(x) = C(-e^{-x}) + 1 - 0 = 1 - Ce^{-x}$ } substitute everywhere in DE for unknown so equation only involves x, then simplify LHS & RHS to see they are same

$\frac{dy}{dx} = x - y \rightarrow \boxed{1 - Ce^{-x} = x - (Ce^{-x} + x - 1)} = x - Ce^{-x} - x + 1 = 1 - Ce^{-x}$ ✓

b) $10 = y(0) = Ce^{-0} + 0 - 1 = C - 1 \rightarrow C = 10 + 1 = 11$

c) $\rightarrow \boxed{y(x) = 11e^{-x} + x - 1}$

$10 \stackrel{?}{=} y(0) = 11e^{-0} + 0 - 1 = 11 - 1 = 10$ ✓