MAT2705-01 08F Test 2 Answers

① $\underbrace{\begin{bmatrix}5\\5\end{bmatrix}}_{\vec{b}} = x_1\begin{bmatrix}2\\-4\end{bmatrix} + x_2\begin{bmatrix}1\\3\end{bmatrix} = \underbrace{\begin{bmatrix}2 & 1\\-4 & 3\end{bmatrix}}_{A}\underbrace{\begin{bmatrix}x_1\\x_2\end{bmatrix}}_{\vec{x}}$

$A^{-1}[A\vec{x}=\vec{b}\,] \rightarrow \vec{x}=A^{-1}\vec{b}$

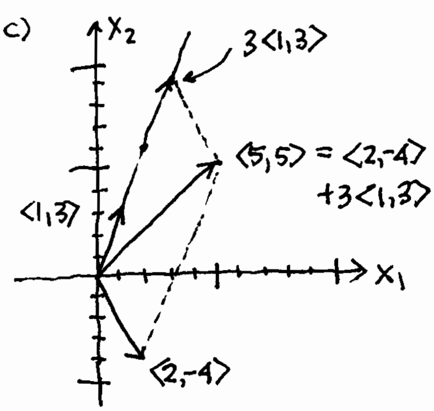
$A^{-1} = \dfrac{1}{6+4}\begin{bmatrix}3 & -1\\4 & 2\end{bmatrix} \qquad \begin{bmatrix}x_1\\x_2\end{bmatrix} = \dfrac{1}{10}\begin{bmatrix}3 & -1\\4 & 2\end{bmatrix}\begin{bmatrix}5\\5\end{bmatrix}$

$= \dfrac{1}{10}\begin{bmatrix}3(5)-5\\4(5)+2(5)\end{bmatrix} = \dfrac{1}{10}\begin{bmatrix}10\\30\end{bmatrix} = \begin{bmatrix}1\\3\end{bmatrix}$

$\therefore \begin{bmatrix}5\\5\end{bmatrix} = \begin{bmatrix}2\\-4\end{bmatrix} + 3\begin{bmatrix}1\\3\end{bmatrix}$

b) $= \begin{bmatrix}2+3\\-4+9\end{bmatrix} = \begin{bmatrix}5\\5\end{bmatrix}$ ✓

c)



② a) $\begin{bmatrix}1 & 1 & 1 & 1\\-1 & 1 & -2 & 2\\1 & 1 & 4 & 0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$

$\langle A|\vec{0}\rangle = \begin{bmatrix}1 & 1 & 1 & 1 & 0\\-1 & 1 & -2 & 2 & 0\\1 & 1 & 4 & 0 & 0\end{bmatrix} \xrightarrow[\text{maple}]{\text{rref}}$

$$\begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & \\ \hline 1 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & +4/3 & 0\\ 0 & 0 & 1 & -1/3 & 0\\ \hline L & L & L & F & \end{array}$$

$\left.\begin{array}{l} x_1 = 0\\ x_2 + 4/3\,x_4 = 0\\ x_3 - 1/3\,x_4 = 0\end{array}\right\}\quad \begin{array}{l} x_1 = 0\\ x_2 = -4/3\,t\\ x_3 = 1/3\,t\end{array}$

$x_4 = t$

$\boxed{\begin{aligned}\langle x_1,x_2,x_3,x_4\rangle &= \langle 0,-4/3\,t, 1/3\,t, t\rangle\\ &= t\langle 0,-4/3,1/3,1\rangle\end{aligned}}$

b) $\boxed{\text{basis of soln space: } \{\langle 0,-4/3,1/3,1\rangle\}}$

②c) $0\begin{bmatrix}1\\-1\\1\end{bmatrix} - \dfrac{4}{3}\begin{bmatrix}1\\1\\1\end{bmatrix} + \dfrac{1}{3}\begin{bmatrix}1\\-2\\4\end{bmatrix} + \begin{bmatrix}1\\2\\0\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$

one independent linear relationship

$= \begin{bmatrix}-4/3 + 1/3 + 1\\-4/3 - 2/3 + 2\\-4/3 + 4/3 + 0\end{bmatrix} = \begin{bmatrix}1-3/3\\2-6/3\\0\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$ ✓

③ $\begin{vmatrix}2 & -3 & -3\\-1 & 1 & 2\\3 & -5 & -4\end{vmatrix} = 0$ linearly dependent

$\begin{vmatrix}1 & 2 & 3\\4 & 5 & 6\\7 & 8 & 9\end{vmatrix} = 0$ linearly dependent

$\begin{vmatrix}1 & 2 & 3\\2 & 3 & 1\\3 & 1 & 2\end{vmatrix} = -18$ linearly independent

determinants evaluated with Maple.

$A\vec{x} = \vec{0}$ with $\det(A) \neq 0$ means

$\vec{x} = A^{-1}\vec{0} = \vec{0}$ only the zero soln exists for coefficients of linear combinations of the columns which equal the zero vector.

When $\det(A) = 0$, nonzero solutions exist, since there must be at least one free variable for the square linear system matrix reduction, hence linear relationships exist among those columns.