

① a) $\frac{dy}{dx} = 3xy^{1/2}$ (separable)

$\int \frac{dy}{y^{1/2}} = \int 3x dx$ (separate)

$\int y^{-1/2} dy = 3 \frac{x^2}{2} + C_1$ (integrate)

$\frac{y^{1/2}}{1/2} = \frac{3}{2}x^2 + C_1$

$y^{1/2} = \frac{1}{2}(\frac{3}{2}x^2 + C_1)$
 $= \frac{3}{4}x^2 + C$

(solve for y, rename $\frac{1}{2}C_1 = C$)

gen soln
 $y = (\frac{3}{4}x^2 + C)^2$
 $= \frac{9}{16}x^4 + \frac{3}{2}x^2C + C^2$

"It takes about 22.2 days to reduce the pollutant concentration from 0.25% to 0.10%."

② b) $\lim_{t \rightarrow \infty} x = \lim_{t \rightarrow \infty} 4 + 16e^{-t/16} = 4$

X approaches 4 asymptotically (from above)

c) $\frac{x(t)}{8000} = .0010 \rightarrow 4 + 16e^{-t/16} = (0.01)(8000) = 8$
 $\frac{4 + 16e^{-t/16}}{8000}$
 $16e^{-t/16} = 4$
 $e^{-t/16} = 1/4$

$-\frac{t}{16} = \ln 1/4 = \ln 4^{-1} = -\ln 4$, $t = -16(-\ln 4) = 16 \ln 4 = 16 \ln 2^2 = 32 \ln 2 \approx 22.1807 \approx 22.2$ days

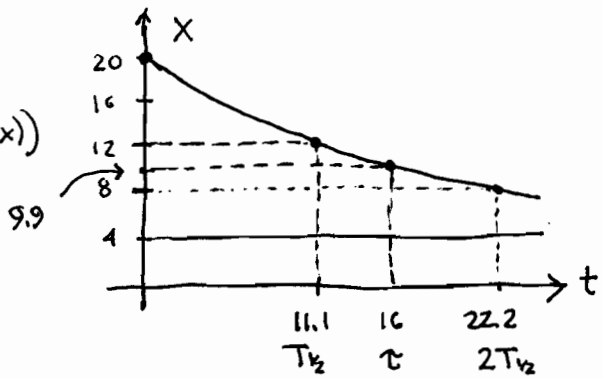
b) $9 = y(0) = C^2 \rightarrow C = 3$
 $\rightarrow 3 = y(0)^{1/2} = C \rightarrow$ positive soln.

Squaring step introduced. Extra solns not allowed: $y(0) = 9 \rightarrow C = \pm 3$

IVP soln
 $y = (\frac{3}{4}x^2 + 3)^2$
 $= \frac{9}{16}(x^2 + 4)^2$

e) $e^{-t/16} \leftarrow \tau = 16$ days "characteristic time"
 $e^{-t/16} = 1/2 \rightarrow -\frac{t}{16} = \ln 2^{-1} = -\ln 2 \rightarrow t = 16 \ln 2 = T_{1/2} \approx 11.090$ "half-life"

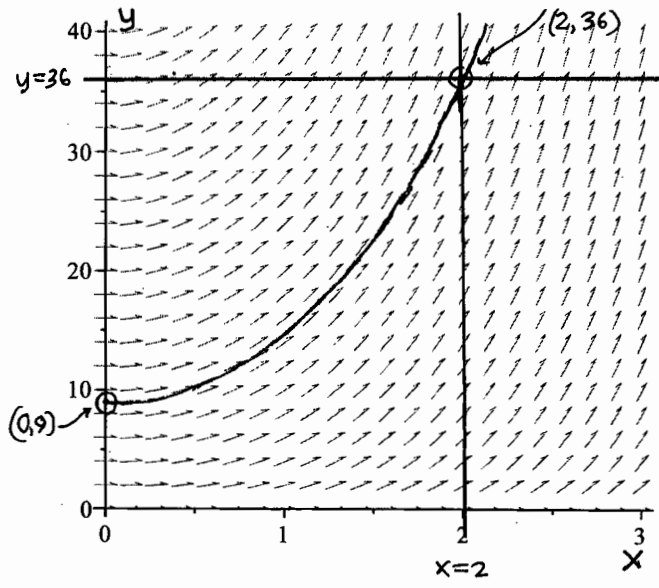
part c) $t = 2$ half-lives, (0.25% reduces by $1/4 = 1/2^2$)



d) $\frac{dy}{dx} = \frac{d}{dx} (\frac{3}{4}x^2 + 3)^2 = 2(\frac{3}{4}x^2 + 3)^1 (\frac{3}{4}(2x)) = 3x(\frac{3}{4}x^2 + 3)$
 $\frac{dy}{dx} = 3xy^{1/2}$
 $3x(\frac{3}{4}x^2 + 3) = 3x(\frac{3}{4}x^2 + 3) \checkmark$

f) $36 = (\frac{3}{4}x^2 + 3)^2$
 $6 = \frac{3}{4}x^2 + 3$
 $3 = \frac{3}{4}x^2$
 $4 = x^2 \rightarrow x = \pm 2 \rightarrow 2$
 since want $x > 0$
 $x = 2$

① e) The graphical soln is dead on, going right through the predicted points.



② a) $\left[\frac{dx}{dt} + \frac{x}{16} = \frac{1}{4} \right] \rightarrow \frac{d}{dt}(xe^{t/16}) = \frac{1}{4}e^{t/16}$
 $\int \frac{1}{16} dt = \frac{t}{16}$
 integrating factor
 $[xe^{t/16} = \frac{1}{4} \frac{e^{t/16}}{1/16} + C] e^{-t/16}$ general soln
 $x = 4 + ce^{-t/16}$
 $20 = x(0) = 4 + C \rightarrow C = 16$
 $x = 4 + 16e^{-t/16}$ IVP soln