MAT2705-01/04 07S Quiz 3 Print Name (Last, First) _____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

$3x_1 + x_2 + x_3 = -5$

$2x_1 + 4x_2 - 6x_3 = 0$

a) Write down the coefficient matrix **A**, the RHS matrix **b** and the augmented matrix **C** = <**A** | **b**> for this linear system of equations.

b) Without technology, hand reduce this matrix to its ReducedRowEchelonForm by hand in 5 easy steps, annotating the MultiplyRow, AddRow, SwapRow operations you apply to each successive matrix in the process, using your own words or the notation: $R_1 \to 3\ R_1, R_1 \to R_1 + 2\ R_1, R_1 \leftrightarrow R_2$.

c) Write out the pair of equations that correspond to the reduced matrix. Identify the leading variables and the free variables and solve for the leading variables, assigning arbitrary parameters to the free variables. State your solution in the form:

$x_1 = \ldots, x_2 = \ldots, x_3 = \ldots$

d) Now check that this solution satisfies the original equations by backsubstituting it into them.

e) Finally check your results with technology, and if incorrect, state the correct results for the reduced matrix and the solution of the system and try to find the error in your hand work (don't erase, cross out).

▶ solution

a) $A = \begin{bmatrix} 3 & 1 & 1 \\ 2 & 4 & -6 \end{bmatrix}$ $b = \begin{bmatrix} -5 \\ 0 \end{bmatrix}$ $C = \begin{bmatrix} 3 & 1 & 1 & -5 \\ 2 & 4 & -6 & 0 \end{bmatrix}$

b) $C \xrightarrow[R_1 \leftrightarrow R_2]{} \begin{bmatrix} 2 & 4 & -6 & 0 \\ 3 & 1 & 1 & -5 \end{bmatrix} \xrightarrow{R_1 \to \frac{1}{2}R_1} \begin{bmatrix} 1 & 2 & -3 & 0 \\ 3 & 1 & 1 & -5 \end{bmatrix} \xrightarrow{R_2 \to R_2 - 3R_1} \begin{bmatrix} 1 & 2 & -3 & 0 \\ 0 & -5 & 10 & -5 \end{bmatrix}$

$\xrightarrow{R_2 \to -\frac{1}{5}R_2} \begin{bmatrix} 1 & 2 & -3 & 0 \\ 0 & 1 & -2 & 1 \end{bmatrix} \xrightarrow{R_1 \to R_1 - 2R_2} \begin{bmatrix} 1 & 0 & 1 & -2 \\ 0 & 1 & -2 & 1 \end{bmatrix}$ done!

c) $x_1 + x_3 = -2 \to x_1 = -2 - x_3 = -2 - t$

$x_2 - 2x_3 = 1 \to x_2 = 1 + 2x_3 = 1 + 2t$

leading free: $x_3 = t$

$x_1 = -2 - t$, $x_2 = 1 + 2t$, $x_3 = t$

d) $3(-2-t) + (1+2t) + (t) = -5$?

$-6 - 3t + 1 + 2t + t = -5$

$-5 = -5$ ✓

$2(-2-t) + 4(1+2t) - 6(t) \stackrel{?}{=} 0$

$-4 - 2t + 4 + 8t - 6t$

$0 + 0 = 0$ ✓

e) > with(Student[LinearAlgebra])

> C := ...

> ReducedRowEchelonForm(C);

Backspace (autocomplete)

yup! but Maple calls $t = -t_1$