

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

1.  $(x^2-1)y' + (x-1)y = 1, y(2) = 1.$

a) First find the general solution of this DE. [Recall  $x^2-1 = (x-1)(x+1)$ !]

b) Then find the solution which satisfies the given initial condition.

c) Now find the solution which satisfies the initial condition  $y(-2) = 0.$

d) ~~Optional No Credit~~: Check that your result for part b) is actually a solution of the DE by backsubstituting.

I forgot to update before printing!

► solution

① a) Put into standard linear form:  $[(x^2-1)\frac{dy}{dx} + (x-1)y = 1] / (x^2-1)$

$$(x+1) \left[ \frac{dy}{dx} + \frac{x-1}{x^2-1} y = \frac{1}{x^2-1} \right] \rightarrow$$

$$\frac{x-1}{(x-1)(x+1)} = \frac{1}{x+1}$$

$$e^{\int \frac{1}{x+1} dx} = \ln|x+1| = |x+1|$$

$$\frac{d}{dx} (y(x+1)) = \frac{x+1}{(x+1)(x-1)} = \frac{1}{x-1}$$

$$[y(x+1) = \int \frac{1}{x-1} dx = \ln|x-1| + C] / (x+1)$$

$$y = \frac{\ln|x-1| + C}{x+1} \quad \text{general soln}$$

abs. value unnecessary since on both sides of D.E., sign irrelevant

b)  $1 = y(2) = \frac{\ln|2-1| + C}{2+1} = \frac{\ln 1 + C}{3} = \frac{C}{3} \rightarrow C = 3 \rightarrow y = \frac{\ln|x-1| + 3}{x+1}$

c)  $0 = y(-2) = \frac{\ln|-2-1| + C}{-2+1} = \frac{\ln 3 + C}{-1} \rightarrow C = -\ln 3 \rightarrow y = \frac{\ln|x-1| - \ln 3}{x+1} = \frac{\ln|\frac{x-1}{3}|}{x+1}$

d)  $\frac{dy}{dx} = \frac{d}{dx} \left( \frac{\ln|x-1| + 3}{x+1} \right) = \frac{(x+1) \left( \frac{1}{x-1} + 0 \right) - (\ln|x-1| + 3)(1+0)}{(x+1)^2}$

$$(x^2-1) \frac{dy}{dx} = (x-1)(x+1) \left[ \frac{1}{(x-1)(x+1)} - \frac{1}{(x+1)^2} (\ln|x-1| + 3) \right]$$

$$= 1 - \left( \frac{x-1}{x+1} \right) (\ln|x-1| + 3)$$

$$(x-1)y = \left( \frac{x-1}{x+1} \right) (\ln|x-1| + 3)$$

$$(x^2-1) \frac{dy}{dx} + (x-1)y = 1 - \left( \frac{x-1}{x+1} \right) (\ln|x-1| + 3) + \left( \frac{x-1}{x+1} \right) (\ln|x-1| + 3) = 1 \quad \checkmark$$

LHS<sub>DE</sub> → 0 RHS<sub>DE</sub>